

Mathematica 11.3 Integration Test Results

Test results for the 594 problems in "4.1.7 (d trig)^m (a+b (c sin)ⁿ)^{p.m"}

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[c+d x]^3}{a-a \operatorname{Sin}[c+d x]^2} d x$$

Optimal (type 3, 58 leaves, 5 steps):

$$-\frac{3 \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{2 a d}+\frac{3 \operatorname{Sec}[c+d x]}{2 a d}-\frac{\operatorname{Csc}[c+d x]^2 \operatorname{Sec}[c+d x]}{2 a d}$$

Result (type 3, 146 leaves):

$$\begin{aligned} & \left(\operatorname{Csc}[c+d x]^4 \left(2-6 \operatorname{Cos}[2 (c+d x)]+2 \operatorname{Cos}[3 (c+d x)]+\right.\right. \\ & 3 \operatorname{Cos}[3 (c+d x)] \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]]-3 \operatorname{Cos}[3 (c+d x)] \operatorname{Log}[\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]]+ \\ & \left.\left.\operatorname{Cos}[c+d x] \left(-2-3 \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]]+3 \operatorname{Log}[\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]]\right)\right)\right) / \\ & \left(2 a d \left(\operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2-\operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2\right)\right) \end{aligned}$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[c+d x]^5}{a-a \operatorname{Sin}[c+d x]^2} d x$$

Optimal (type 3, 82 leaves, 6 steps):

$$\begin{aligned} & -\frac{15 \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{8 a d}+\frac{15 \operatorname{Sec}[c+d x]}{8 a d}- \\ & \frac{5 \operatorname{Csc}[c+d x]^2 \operatorname{Sec}[c+d x]}{8 a d}-\frac{\operatorname{Csc}[c+d x]^4 \operatorname{Sec}[c+d x]}{4 a d} \end{aligned}$$

Result (type 3, 194 leaves):

$$\frac{1}{a} \left(-\frac{7 \csc\left[\frac{1}{2} (c + d x)\right]^2}{32 d} - \frac{\csc\left[\frac{1}{2} (c + d x)\right]^4}{64 d} - \frac{15 \log[\cos\left[\frac{1}{2} (c + d x)\right]]}{8 d} + \right.$$

$$\frac{15 \log[\sin\left[\frac{1}{2} (c + d x)\right]]}{8 d} + \frac{7 \sec\left[\frac{1}{2} (c + d x)\right]^2}{32 d} + \frac{\sec\left[\frac{1}{2} (c + d x)\right]^4}{64 d} +$$

$$\left. \frac{\sin\left[\frac{1}{2} (c + d x)\right]}{d (\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right])} - \frac{\sin\left[\frac{1}{2} (c + d x)\right]}{d (\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right])} \right)$$

Problem 54: Result more than twice size of optimal antiderivative.

$$\int \frac{\csc[c + d x]^3}{(a - a \sin[c + d x]^2)^2} dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$-\frac{5 \operatorname{ArcTanh}[\cos[c + d x]]}{2 a^2 d} + \frac{5 \sec[c + d x]}{2 a^2 d} + \frac{5 \sec[c + d x]^3}{6 a^2 d} - \frac{\csc[c + d x]^2 \sec[c + d x]^3}{2 a^2 d}$$

Result (type 3, 208 leaves):

$$\frac{1}{3 a^2 d \left(\csc\left[\frac{1}{2} (c + d x)\right]^2 - \sec\left[\frac{1}{2} (c + d x)\right]^2\right)^3} 2 \csc[c + d x]^8$$

$$\left(22 - 40 \cos[2 (c + d x)] + 13 \cos[3 (c + d x)] - 30 \cos[4 (c + d x)] + 13 \cos[5 (c + d x)] + \right.$$

$$15 \cos[3 (c + d x)] \log[\cos[\frac{1}{2} (c + d x)]] + 15 \cos[5 (c + d x)] \log[\cos[\frac{1}{2} (c + d x)]] -$$

$$15 \cos[3 (c + d x)] \log[\sin[\frac{1}{2} (c + d x)]] - 15 \cos[5 (c + d x)] \log[\sin[\frac{1}{2} (c + d x)]] +$$

$$\left. \cos[c + d x] \left(-26 - 30 \log[\cos[\frac{1}{2} (c + d x)]] + 30 \log[\sin[\frac{1}{2} (c + d x)]] \right) \right)$$

Problem 66: Result more than twice size of optimal antiderivative.

$$\int \csc[c + d x] (a + b \sin[c + d x]^2) dx$$

Optimal (type 3, 26 leaves, 2 steps):

$$-\frac{a \operatorname{ArcTanh}[\cos[c + d x]]}{d} - \frac{b \cos[c + d x]}{d}$$

Result (type 3, 63 leaves):

$$-\frac{b \cos[c] \cos[d x]}{d} - \frac{a \log[\cos[\frac{c}{2} + \frac{d x}{2}]]}{d} + \frac{a \log[\sin[\frac{c}{2} + \frac{d x}{2}]]}{d} + \frac{b \sin[c] \sin[d x]}{d}$$

Problem 67: Result more than twice size of optimal antiderivative.

$$\int \csc[c + d x]^3 (a + b \sin[c + d x]^2) dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$-\frac{(a+2b) \operatorname{ArcTanh}[\cos[c+d x]]}{2 d} - \frac{a \cot[c+d x] \csc[c+d x]}{2 d}$$

Result (type 3, 118 leaves):

$$-\frac{a \csc\left[\frac{1}{2} (c+d x)\right]^2}{8 d} - \frac{b \log[\cos\left[\frac{c}{2} + \frac{d x}{2}\right]]}{d} - \frac{a \log[\cos\left[\frac{1}{2} (c+d x)\right]]}{2 d} + \\ \frac{b \log[\sin\left[\frac{c}{2} + \frac{d x}{2}\right]]}{d} + \frac{a \log[\sin\left[\frac{1}{2} (c+d x)\right]]}{2 d} + \frac{a \sec\left[\frac{1}{2} (c+d x)\right]^2}{8 d}$$

Problem 76: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + b \sin[x]^2)^3 dx$$

Optimal (type 3, 87 leaves, 2 steps):

$$\frac{1}{16} (2 a + b) (8 a^2 + 8 a b + 5 b^2) x - \frac{1}{48} b (64 a^2 + 54 a b + 15 b^2) \cos[x] \sin[x] - \\ \frac{5}{24} b^2 (2 a + b) \cos[x] \sin[x]^3 - \frac{1}{6} b \cos[x] \sin[x] (a + b \sin[x]^2)^2$$

Result (type 3, 80 leaves):

$$\frac{1}{192} (12 (2 a + b) (8 a^2 + 8 a b + 5 b^2) x + \\ 9 i b (4 i a + (1 + 2 i) b) (4 a + (2 + i) b) \sin[2 x] + 9 b^2 (2 a + b) \sin[4 x] - b^3 \sin[6 x])$$

Problem 78: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin[c + d x]^7}{a + b \sin[c + d x]^2} dx$$

Optimal (type 3, 106 leaves, 4 steps):

$$\frac{a^3 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \cos[c+d x]}{\sqrt{a+b}}\right]}{b^{7/2} \sqrt{a+b} d} - \frac{(a^2 - a b + b^2) \cos[c+d x]}{b^3 d} - \frac{(a - 2 b) \cos[c+d x]^3}{3 b^2 d} - \frac{\cos[c+d x]^5}{5 b d}$$

Result (type 3, 180 leaves):

$$\left(-240 a^3 \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right] - 240 a^3 \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right] - 2 \sqrt{-a - b} \sqrt{b} \cos [c + d x] (120 a^2 - 100 a b + 89 b^2 + 4 (5 a - 7 b) b \cos [2 (c + d x)] + 3 b^2 \cos [4 (c + d x)]) \right) / (240 \sqrt{-a - b} b^{7/2} d)$$

Problem 79: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin [c + d x]^5}{a + b \sin [c + d x]^2} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$-\frac{a^2 \operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos [c + d x]}{\sqrt{a + b}} \right]}{b^{5/2} \sqrt{a + b} d} + \frac{(a - b) \cos [c + d x]}{b^2 d} + \frac{\cos [c + d x]^3}{3 b d}$$

Result (type 3, 150 leaves):

$$\frac{1}{6 \sqrt{-a - b} b^{5/2} d} \left(6 a^2 \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right] + 6 a^2 \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right] + \sqrt{-a - b} \sqrt{b} \cos [c + d x] (6 a - 5 b + b \cos [2 (c + d x)]) \right)$$

Problem 80: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sin [c + d x]^3}{a + b \sin [c + d x]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$\frac{a \operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos [c + d x]}{\sqrt{a + b}} \right]}{b^{3/2} \sqrt{a + b} d} - \frac{\cos [c + d x]}{b d}$$

Result (type 3, 125 leaves):

$$-\frac{1}{\sqrt{-a-b} b^{3/2} d} \left(a \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a-b}} \right] + a \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a-b}} \right] + \sqrt{-a-b} \sqrt{b} \cos [c + d x] \right)$$

Problem 81: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sin [c + d x]}{a + b \sin [c + d x]^2} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos [c+d x]}{\sqrt{a+b}} \right]}{\sqrt{b} \sqrt{a+b} d}$$

Result (type 3, 97 leaves):

$$\frac{\operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a-b}} \right] + \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a-b}} \right]}{\sqrt{-a-b} \sqrt{b} d}$$

Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\csc [c + d x]}{a + b \sin [c + d x]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh} [\cos [c + d x]]}{a d} + \frac{\sqrt{b} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos [c+d x]}{\sqrt{a+b}} \right]}{a \sqrt{a+b} d}$$

Result (type 3, 143 leaves):

$$-\frac{1}{a d} \left(\frac{\sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a-b}} \right]}{\sqrt{-a-b}} + \frac{\sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a-b}} \right]}{\sqrt{-a-b}} + \operatorname{Log} [\cos \left[\frac{1}{2} (c + d x) \right]] - \operatorname{Log} [\sin \left[\frac{1}{2} (c + d x) \right]] \right)$$

Problem 83: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[c+d x]^3}{a+b \operatorname{Sin}[c+d x]^2} d x$$

Optimal (type 3, 85 leaves, 5 steps):

$$-\frac{(a-2 b) \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{2 a^2 d}-\frac{b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Cos}[c+d x]}{\sqrt{a+b}}\right]}{a^2 \sqrt{a+b} d}-\frac{\operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]}{2 a d}$$

Result (type 3, 224 leaves):

$$\begin{aligned} & -\left(\left(\left(2 a+b-b \operatorname{Cos}[2(c+d x)]\right) \operatorname{Csc}[c+d x]^2\left(-8 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}-i \sqrt{a} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{-a-b}}\right]\right.\right.\right. \\ & \quad \left.\left.\left.8 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}+i \sqrt{a} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{-a-b}}\right]+\sqrt{-a-b}\right.\right.\right. \\ & \quad \left.\left.\left.\left(a \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2+4(a-2 b)\left(\operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]]-\operatorname{Log}[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]]\right)-\right.\right.\right. \\ & \quad \left.\left.\left.a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right)\right)\right)\right) / \left(16 a^2 \sqrt{-a-b} d(b+a \operatorname{Csc}[c+d x]^2)\right) \end{aligned}$$

Problem 84: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[c+d x]^5}{a+b \operatorname{Sin}[c+d x]^2} d x$$

Optimal (type 3, 125 leaves, 6 steps):

$$\begin{aligned} & -\frac{(3 a^2-4 a b+8 b^2) \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{8 a^3 d}+\frac{b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Cos}[c+d x]}{\sqrt{a+b}}\right]}{a^3 \sqrt{a+b} d}- \\ & \frac{(3 a-4 b) \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]}{8 a^2 d}-\frac{\operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^3}{4 a d} \end{aligned}$$

Result (type 3, 657 leaves):

$$\begin{aligned}
& \left(b^{5/2} \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right] \left(\sqrt{b} \cos \left[\frac{1}{2} (c + d x) \right] - i \sqrt{a} \sin \left[\frac{1}{2} (c + d x) \right] \right)}{\sqrt{-a - b}} \right] \right. \\
& \quad \left. (-2 a - b + b \cos \left[2 (c + d x) \right]) \csc^2(c + d x) \right) / \left(2 a^3 \sqrt{-a - b} d (b + a \csc^2(c + d x)) \right) + \\
& \left(b^{5/2} \operatorname{ArcTan} \left[\frac{\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right] \left(\sqrt{b} \cos \left[\frac{1}{2} (c + d x) \right] + i \sqrt{a} \sin \left[\frac{1}{2} (c + d x) \right] \right)}{\sqrt{-a - b}} \right] \right. \\
& \quad \left. (-2 a - b + b \cos \left[2 (c + d x) \right]) \csc^2(c + d x) \right) / \left(2 a^3 \sqrt{-a - b} d (b + a \csc^2(c + d x)) \right) + \\
& \left((3 a - 4 b) (-2 a - b + b \cos \left[2 (c + d x) \right]) \csc^2 \left[\frac{1}{2} (c + d x) \right]^2 \csc^2(c + d x) \right) / \\
& \quad (64 a^2 d (b + a \csc^2(c + d x))) + \\
& \frac{(-2 a - b + b \cos \left[2 (c + d x) \right]) \csc^2 \left[\frac{1}{2} (c + d x) \right]^4 \csc^2(c + d x)}{128 a d (b + a \csc^2(c + d x))} + \\
& \left((3 a^2 - 4 a b + 8 b^2) (-2 a - b + b \cos \left[2 (c + d x) \right]) \csc^2(c + d x)^2 \log[\cos \left[\frac{1}{2} (c + d x) \right]] \right) / \\
& \quad (16 a^3 d (b + a \csc^2(c + d x))) + \\
& \left((-3 a^2 + 4 a b - 8 b^2) (-2 a - b + b \cos \left[2 (c + d x) \right]) \csc^2(c + d x)^2 \log[\sin \left[\frac{1}{2} (c + d x) \right]] \right) / \\
& \quad (16 a^3 d (b + a \csc^2(c + d x))) + \\
& \left((-3 a + 4 b) (-2 a - b + b \cos \left[2 (c + d x) \right]) \csc^2(c + d x)^2 \sec^2 \left[\frac{1}{2} (c + d x) \right]^2 \right) / \\
& \quad (64 a^2 d (b + a \csc^2(c + d x))) - \frac{(-2 a - b + b \cos \left[2 (c + d x) \right]) \csc^2(c + d x)^2 \sec^2 \left[\frac{1}{2} (c + d x) \right]^4}{128 a d (b + a \csc^2(c + d x))}
\end{aligned}$$

Problem 94: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin(c + d x)^7}{(a + b \sin(c + d x)^2)^2} dx$$

Optimal (type 3, 128 leaves, 5 steps):

$$\begin{aligned}
& -\frac{a^2 (5 a + 6 b) \operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos(c + d x)}{\sqrt{a + b}} \right]}{2 b^{7/2} (a + b)^{3/2} d} + \frac{(2 a - b) \cos(c + d x)}{b^3 d} + \\
& \frac{\cos(c + d x)^3}{3 b^2 d} + \frac{a^3 \cos(c + d x)}{2 b^3 (a + b) d (a + b - b \cos(c + d x)^2)}
\end{aligned}$$

Result (type 3, 194 leaves):

$$\begin{aligned} & \frac{1}{12 b^{7/2} d} \\ & - \frac{6 a^2 (5 a + 6 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a-b}} \right]}{(-a-b)^{3/2}} - \frac{6 a^2 (5 a + 6 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a-b}} \right]}{(-a-b)^{3/2}} + \\ & \sqrt{b} \left(\cos [c+d x] \left(24 a - 9 b + \frac{12 a^3}{(a+b) (2 a + b - b \cos [2 (c+d x)])} \right) + b \cos [3 (c+d x)] \right) \end{aligned}$$

Problem 95: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin [c+d x]^5}{(a+b \sin [c+d x]^2)^2} dx$$

Optimal (type 3, 102 leaves, 5 steps) :

$$\frac{a (3 a + 4 b) \operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos [c+d x]}{\sqrt{a+b}} \right]}{2 b^{5/2} (a+b)^{3/2} d} - \frac{\cos [c+d x]}{b^2 d} - \frac{a^2 \cos [c+d x]}{2 b^2 (a+b) d (a+b - b \cos [c+d x]^2)}$$

Result (type 3, 172 leaves) :

$$\begin{aligned} & \frac{1}{2 b^{5/2} d} \left(\frac{a (3 a + 4 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a-b}} \right]}{(-a-b)^{3/2}} + \frac{a (3 a + 4 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a-b}} \right]}{(-a-b)^{3/2}} + \right. \\ & \left. 2 \sqrt{b} \cos [c+d x] \left(-1 - \frac{a^2}{(a+b) (2 a + b - b \cos [2 (c+d x)])} \right) \right) \end{aligned}$$

Problem 96: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin [c+d x]^3}{(a+b \sin [c+d x]^2)^2} dx$$

Optimal (type 3, 83 leaves, 3 steps) :

$$-\frac{(a+2 b) \operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos [c+d x]}{\sqrt{a+b}} \right]}{2 b^{3/2} (a+b)^{3/2} d} + \frac{a \cos [c+d x]}{2 b (a+b) d (a+b - b \cos [c+d x]^2)}$$

Result (type 3, 160 leaves) :

$$\frac{1}{2 b^{3/2} (a + b) d} \left(\frac{(a + 2 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right]}{\sqrt{-a - b}} + \frac{(a + 2 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right]}{\sqrt{-a - b}} + \frac{2 a \sqrt{b} \cos [c + d x]}{2 a + b - b \cos [2 (c + d x)]} \right)$$

Problem 97: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sin [c + d x]}{(a + b \sin [c + d x]^2)^2} dx$$

Optimal (type 3, 74 leaves, 3 steps) :

$$-\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos [c + d x]}{\sqrt{a + b}} \right]}{2 \sqrt{b} (a + b)^{3/2} d} - \frac{\cos [c + d x]}{2 (a + b) d (a + b - b \cos [c + d x]^2)}$$

Result (type 3, 149 leaves) :

$$\frac{1}{2 (a + b) d} \left(\frac{\operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right]}{\sqrt{-a - b} \sqrt{b}} + \frac{\operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right]}{\sqrt{-a - b} \sqrt{b}} - \frac{2 \cos [c + d x]}{2 a + b - b \cos [2 (c + d x)]} \right)$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\csc [c + d x]}{(a + b \sin [c + d x]^2)^2} dx$$

Optimal (type 3, 103 leaves, 5 steps) :

$$-\frac{\operatorname{ArcTanh} [\cos [c + d x]]}{a^2 d} + \frac{\sqrt{b} (3 a + 2 b) \operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos [c + d x]}{\sqrt{a + b}} \right]}{2 a^2 (a + b)^{3/2} d} + \frac{b \cos [c + d x]}{2 a (a + b) d (a + b - b \cos [c + d x]^2)}$$

Result (type 3, 194 leaves) :

$$\frac{1}{2 a^2 d} \left(\frac{\sqrt{b} (3 a + 2 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right]}{(-a - b)^{3/2}} + \frac{\sqrt{b} (3 a + 2 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right]}{(-a - b)^{3/2}} + \right. \\ \left. 2 \left(\frac{a b \cos [c + d x]}{(a + b) (2 a + b - b \cos [2 (c + d x)])} - \operatorname{Log} [\cos [\frac{1}{2} (c + d x)]] + \operatorname{Log} [\sin [\frac{1}{2} (c + d x)]] \right) \right)$$

Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\csc [c + d x]^3}{(a + b \sin [c + d x]^2)^2} dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$-\frac{(a - 4 b) \operatorname{ArcTanh} [\cos [c + d x]]}{2 a^3 d} - \frac{b^{3/2} (5 a + 4 b) \operatorname{ArcTanh} \left[\frac{\sqrt{b} \cos [c + d x]}{\sqrt{a + b}} \right]}{2 a^3 (a + b)^{3/2} d} - \\ \frac{b (a + 2 b) \cos [c + d x]}{2 a^2 (a + b) d (a + b - b \cos [c + d x]^2)} - \frac{\cot [c + d x] \csc [c + d x]}{2 a d (a + b - b \cos [c + d x]^2)}$$

Result (type 3, 390 leaves):

$$\frac{1}{32 a^3 d (b + a \csc [c + d x]^2)^2} \\ (-2 a - b + b \cos [2 (c + d x)]) \csc [c + d x]^3 \left(\frac{8 a b^2 \cot [c + d x]}{a + b} + \frac{1}{(-a - b)^{3/2}} 4 b^{3/2} (5 a + 4 b) \right. \\ \left. \operatorname{ArcTan} \left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right] (2 a + b - b \cos [2 (c + d x)]) \csc [c + d x] + \frac{1}{(-a - b)^{3/2}} \right. \\ \left. 4 b^{3/2} (5 a + 4 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-a - b}} \right] (2 a + b - b \cos [2 (c + d x)]) \right. \\ \left. \csc [c + d x] + a (2 a + b - b \cos [2 (c + d x)]) \csc \left[\frac{1}{2} (c + d x) \right]^2 \csc [c + d x] + \right. \\ \left. 4 (a - 4 b) (2 a + b - b \cos [2 (c + d x)]) \csc [c + d x] \operatorname{Log} [\cos [\frac{1}{2} (c + d x)]] - \right. \\ \left. 4 (a - 4 b) (2 a + b - b \cos [2 (c + d x)]) \csc [c + d x] \operatorname{Log} [\sin [\frac{1}{2} (c + d x)]] - \right. \\ \left. a (2 a + b - b \cos [2 (c + d x)]) \csc [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right)$$

Problem 113: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{\sin[x]}{\sqrt{1 + \sin[x]^2}} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$-\text{ArcSin}\left[\frac{\cos[x]}{\sqrt{2}}\right]$$

Result (type 3, 29 leaves):

$$i \log[i \sqrt{2} \cos[x] + \sqrt{3 - \cos[2x]}]$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int \sin[x] \sqrt{1 + \sin[x]^2} dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$-\text{ArcSin}\left[\frac{\cos[x]}{\sqrt{2}}\right] - \frac{1}{2} \cos[x] \sqrt{2 - \cos[x]^2}$$

Result (type 3, 53 leaves):

$$-\frac{\cos[x] \sqrt{3 - \cos[2x]}}{2\sqrt{2}} + i \log[i \sqrt{2} \cos[x] + \sqrt{3 - \cos[2x]}]$$

Problem 115: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sin[7+3x]}{\sqrt{3+\sin[7+3x]^2}} dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$-\frac{1}{3} \text{ArcSin}\left[\frac{1}{2} \cos[7+3x]\right]$$

Result (type 3, 39 leaves):

$$\frac{1}{3} i \log[i \sqrt{2} \cos[7+3x] + \sqrt{7 - \cos[2(7+3x)]}]$$

Problem 119: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a - a \sin[x]^2}} dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$\frac{\text{ArcTanh}[\sin[x] \cos[x]]}{\sqrt{a \cos[x]^2}}$$

Result (type 3, 46 leaves):

$$\frac{\cos[x] \left(-\log[\cos[\frac{x}{2}] - \sin[\frac{x}{2}]] + \log[\cos[\frac{x}{2}] + \sin[\frac{x}{2}]] \right)}{\sqrt{a \cos[x]^2}}$$

Problem 120: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a - a \sin[x]^2)^{3/2}} dx$$

Optimal (type 3, 42 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\sin[x] \cos[x]]}{2 a \sqrt{a \cos[x]^2}} + \frac{\tan[x]}{2 a \sqrt{a \cos[x]^2}}$$

Result (type 3, 91 leaves):

$$-\frac{1}{4 (a \cos[x]^2)^{3/2}} + \cos[x] \left(\log[\cos[\frac{x}{2}] - \sin[\frac{x}{2}]] + \cos[2x] \left(\log[\cos[\frac{x}{2}] - \sin[\frac{x}{2}]] - \log[\cos[\frac{x}{2}] + \sin[\frac{x}{2}]] \right) - \log[\cos[\frac{x}{2}] + \sin[\frac{x}{2}]] - 2 \sin[x] \right)$$

Problem 172: Result unnecessarily involves higher level functions.

$$\int \sin[e + fx]^5 (a + b \sin[e + fx]^2)^p dx$$

Optimal (type 5, 220 leaves, 5 steps):

$$\begin{aligned} & \frac{(3a - 2b(2+p)) \cos[e+fx] (a+b - b \cos[e+fx]^2)^{1+p}}{b^2 f (3+2p) (5+2p)} - \\ & \left((3a^2 - 4ab(1+p) + 4b^2(2+3p+p^2)) \cos[e+fx] (a+b - b \cos[e+fx]^2)^p \right. \\ & \left. \left(1 - \frac{b \cos[e+fx]^2}{a+b} \right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{b \cos[e+fx]^2}{a+b}\right] \right) / \\ & (b^2 f (3+2p) (5+2p)) - \frac{\cos[e+fx] (a+b - b \cos[e+fx]^2)^{1+p} \sin[e+fx]^2}{b f (5+2p)} \end{aligned}$$

Result (type 6, 184 leaves):

$$\left(4 a \text{AppellF1}\left[3, \frac{1}{2}, -p, 4, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] \right. \\ \left. \sin[e+f x]^5 (a+b \sin[e+f x]^2)^p \tan[e+f x]\right) / \\ \left(3 f \left(8 a \text{AppellF1}\left[3, \frac{1}{2}, -p, 4, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] + \right. \right. \\ \left. \left. 2 b p \text{AppellF1}\left[4, \frac{1}{2}, 1-p, 5, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] + \right. \right. \\ \left. \left. a \text{AppellF1}\left[4, \frac{3}{2}, -p, 5, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right]\right) \sin[e+f x]^2 \right)$$

Problem 173: Result unnecessarily involves higher level functions.

$$\int \sin[e+f x]^3 (a+b \sin[e+f x]^2)^p dx$$

Optimal (type 5, 131 leaves, 4 steps):

$$-\frac{\cos[e+f x] (a+b-b \cos[e+f x]^2)^{1+p}}{b f (3+2 p)} + \frac{1}{b f (3+2 p)} (a-2 b (1+p)) \cos[e+f x] \\ (a+b-b \cos[e+f x]^2)^p \left(1 - \frac{b \cos[e+f x]^2}{a+b}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{b \cos[e+f x]^2}{a+b}\right]$$

Result (type 6, 184 leaves):

$$\left(3 a \text{AppellF1}\left[2, \frac{1}{2}, -p, 3, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] \right. \\ \left. \sin[e+f x]^3 (a+b \sin[e+f x]^2)^p \tan[e+f x]\right) / \\ \left(2 f \left(6 a \text{AppellF1}\left[2, \frac{1}{2}, -p, 3, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] + \right. \right. \\ \left. \left. 2 b p \text{AppellF1}\left[3, \frac{1}{2}, 1-p, 4, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] + \right. \right. \\ \left. \left. a \text{AppellF1}\left[3, \frac{3}{2}, -p, 4, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right]\right) \sin[e+f x]^2 \right)$$

Problem 175: Unable to integrate problem.

$$\int \csc[e+f x] (a+b \sin[e+f x]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$-\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, \cos[e+f x]^2, \frac{b \cos[e+f x]^2}{a+b}\right] \\ \cos[e+f x] (a+b-b \cos[e+f x]^2)^p \left(1 - \frac{b \cos[e+f x]^2}{a+b}\right)^{-p}$$

Result (type 8, 23 leaves):

$$\int \csc[e + fx] (a + b \sin[e + fx]^2)^p dx$$

Problem 176: Unable to integrate problem.

$$\int \csc[e + fx]^3 (a + b \sin[e + fx]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\begin{aligned} & -\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, \cos[e + fx]^2, \frac{b \cos[e + fx]^2}{a + b}\right] \\ & \cos[e + fx] (a + b - b \cos[e + fx]^2)^p \left(1 - \frac{b \cos[e + fx]^2}{a + b}\right)^{-p} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \csc[e + fx]^3 (a + b \sin[e + fx]^2)^p dx$$

Problem 177: Unable to integrate problem.

$$\int \csc[e + fx]^5 (a + b \sin[e + fx]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\begin{aligned} & -\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 3, -p, \frac{3}{2}, \cos[e + fx]^2, \frac{b \cos[e + fx]^2}{a + b}\right] \\ & \cos[e + fx] (a + b - b \cos[e + fx]^2)^p \left(1 - \frac{b \cos[e + fx]^2}{a + b}\right)^{-p} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \csc[e + fx]^5 (a + b \sin[e + fx]^2)^p dx$$

Problem 179: Result more than twice size of optimal antiderivative.

$$\int \sin[e + fx]^2 (a + b \sin[e + fx]^2)^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\begin{aligned} & \frac{1}{3f} \text{AppellF1}\left[\frac{3}{2}, 2+p, -p, \frac{5}{2}, -\tan[e + fx]^2, -\frac{(a+b) \tan[e + fx]^2}{a}\right] \\ & (\sec[e + fx]^2)^p (a + b \sin[e + fx]^2)^p \tan[e + fx]^3 \left(1 + \frac{(a+b) \tan[e + fx]^2}{a}\right)^{-p} \end{aligned}$$

Result (type 6, 240 leaves):

$$\begin{aligned}
& - \left(\left(2^{-2-p} \sqrt{\frac{b \cos[e+f x]^2}{a+b}} (2 a + b - b \cos[2(e+f x)])^{1+p} \right. \right. \\
& \quad \left(2 a (2+p) \text{AppellF1}[1+p, \frac{1}{2}, \frac{1}{2}, 2+p, \frac{2 a + b - b \cos[2(e+f x)]}{2(a+b)}, \right. \\
& \quad \left. \frac{2 a + b - b \cos[2(e+f x)]}{2 a}] - (1+p) \text{AppellF1}[2+p, \frac{1}{2}, \frac{1}{2}, 3+p, \right. \\
& \quad \left. \frac{2 a + b - b \cos[2(e+f x)]}{2(a+b)}, \frac{2 a + b - b \cos[2(e+f x)]}{2 a}] (2 a + b - b \cos[2(e+f x)]) \right) \\
& \quad \left. \csc[2(e+f x)] \sqrt{-\frac{b \sin[e+f x]^2}{a}} \right) / (b^2 f (1+p) (2+p))
\end{aligned}$$

Problem 180: Unable to integrate problem.

$$\int \csc[e+f x]^2 (a+b \sin[e+f x]^2)^p dx$$

Optimal (type 6, 97 leaves, 3 steps):

$$\begin{aligned}
& - \frac{1}{f} \text{AppellF1}\left[-\frac{1}{2}, \frac{1}{2}, -p, \frac{1}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] \\
& \quad \sqrt{\cos[e+f x]^2} \csc[e+f x] \sec[e+f x] (a+b \sin[e+f x]^2)^p \left(1 + \frac{b \sin[e+f x]^2}{a}\right)^{-p}
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int \csc[e+f x]^2 (a+b \sin[e+f x]^2)^p dx$$

Problem 181: Unable to integrate problem.

$$\int \csc[e+f x]^4 (a+b \sin[e+f x]^2)^p dx$$

Optimal (type 6, 101 leaves, 3 steps):

$$\begin{aligned}
& - \frac{1}{3 f} \text{AppellF1}\left[-\frac{3}{2}, \frac{1}{2}, -p, -\frac{1}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] \\
& \quad \sqrt{\cos[e+f x]^2} \csc[e+f x]^3 \sec[e+f x] (a+b \sin[e+f x]^2)^p \left(1 + \frac{b \sin[e+f x]^2}{a}\right)^{-p}
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int \csc[e+f x]^4 (a+b \sin[e+f x]^2)^p dx$$

Problem 182: Result is not expressed in closed-form.

$$\int \frac{\sin[c + dx]^7}{a + b \sin[c + dx]^3} dx$$

Optimal (type 3, 335 leaves, 17 steps):

$$\begin{aligned} & \frac{3x}{8b} + \frac{2(-1)^{2/3} a^{5/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} b^{7/3} d} - \\ & \frac{2a^{5/3} \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3\sqrt{a^{2/3} - b^{2/3}} b^{7/3} d} + \frac{2(-1)^{1/3} a^{5/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} b^{7/3} d} + \\ & \frac{a \cos[c + dx]}{b^2 d} - \frac{3 \cos[c + dx] \sin[c + dx]}{8bd} - \frac{\cos[c + dx] \sin[c + dx]^3}{4bd} \end{aligned}$$

Result (type 7, 219 leaves):

$$\begin{aligned} & \frac{1}{96b^2 d} \left(96a \cos[c + dx] - 32a^2 \operatorname{RootSum}\left[-\frac{1}{2}b + 3\frac{1}{2}b\#\mathbb{1}^2 + 8a\#\mathbb{1}^3 - 3\frac{1}{2}b\#\mathbb{1}^4 + \frac{1}{2}b\#\mathbb{1}^6 \&, \right. \right. \\ & \left. \left. \left(-2\operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#\mathbb{1}}\right] + \frac{1}{2}\operatorname{Log}\left[1 - 2\cos[c + dx]\#\mathbb{1} + \#\mathbb{1}^2\right] + 2\operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#\mathbb{1}}\right] \right. \right. \\ & \left. \left. \left. \#\mathbb{1}^2 - \frac{1}{2}\operatorname{Log}\left[1 - 2\cos[c + dx]\#\mathbb{1} + \#\mathbb{1}^2\right]\#\mathbb{1}^2 \right) \right/ (b - 4\frac{1}{2}a\#\mathbb{1} - 2b\#\mathbb{1}^2 + b\#\mathbb{1}^4) \& \right] + \\ & 3b(12(c + dx) - 8\sin[2(c + dx)] + \sin[4(c + dx)]) \right) \end{aligned}$$

Problem 183: Result is not expressed in closed-form.

$$\int \frac{\sin[c + dx]^5}{a + b \sin[c + dx]^3} dx$$

Optimal (type 3, 273 leaves, 15 steps):

$$\begin{aligned} & \frac{x}{2b} - \frac{2a \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3\sqrt{a^{2/3} - b^{2/3}} b^{5/3} d} + \frac{2a \operatorname{ArcTanh}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}}\right]}{3\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} b^{5/3} d} + \\ & \frac{2a \operatorname{ArcTanh}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}}\right]}{3\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} b^{5/3} d} - \frac{\cos[c + dx] \sin[c + dx]}{2bd} \end{aligned}$$

Result (type 7, 255 leaves):

$$\frac{1}{12 b d} \left(6 (c + d x) - 2 i a \operatorname{RootSum} \left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \right. \right.$$

$$\left(2 \operatorname{ArcTan} \left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1} \right] - i \operatorname{Log} \left[1 - 2 \cos[c + d x] \#1 + \#1^2 \right] - \right.$$

$$\left. 4 \operatorname{ArcTan} \left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1} \right] \#1^2 + 2 i \operatorname{Log} \left[1 - 2 \cos[c + d x] \#1 + \#1^2 \right] \#1^2 + \right.$$

$$\left. 2 \operatorname{ArcTan} \left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1} \right] \#1^4 - i \operatorname{Log} \left[1 - 2 \cos[c + d x] \#1 + \#1^2 \right] \#1^4 \right) \Big/$$

$$\left. \left(b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5 \right) \& \right] - 3 \sin[2 (c + d x)] \Big)$$

Problem 184: Result is not expressed in closed-form.

$$\int \frac{\sin[c + d x]^3}{a + b \sin[c + d x]^3} dx$$

Optimal (type 3, 259 leaves, 13 steps):

$$\frac{x}{b} - \frac{2 a^{1/3} \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} b d} -$$

$$\frac{2 a^{1/3} \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} b d} + \frac{2 a^{1/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/3} (b^{1/3} + (-1)^{2/3} a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right])}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} b d}$$

Result (type 7, 140 leaves):

$$\frac{1}{3 b d} \left(3 c + 3 d x + 2 i a \operatorname{RootSum} \left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \right. \right.$$

$$\left. \left. \frac{2 \operatorname{ArcTan} \left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1} \right] \#1 - i \operatorname{Log} \left[1 - 2 \cos[c + d x] \#1 + \#1^2 \right] \#1}{b - 4 i a \#1 - 2 b \#1^2 + b \#1^4} \& \right] \right)$$

Problem 185: Result is not expressed in closed-form.

$$\int \frac{\sin[c + d x]}{a + b \sin[c + d x]^3} dx$$

Optimal (type 3, 267 leaves, 11 steps):

$$\frac{2 (-1)^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3}-a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}}}\right]}{3 a^{1/3} \sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}} b^{1/3} d} -$$

$$\frac{2 \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3 a^{1/3} \sqrt{a^{2/3}-b^{2/3}} b^{1/3} d} + \frac{2 (-1)^{1/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3}+a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}}}\right]}{3 a^{1/3} \sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}} b^{1/3} d}$$

Result (type 7, 172 leaves) :

$$-\frac{1}{3 d} \operatorname{RootSum}\left[-\frac{1}{2} b+3 \frac{1}{2} b \#1^2+8 a \#1^3-3 \frac{1}{2} b \#1^4+\frac{1}{2} b \#1^6 \&, \right. \\ \left. -2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]+\frac{1}{2} \log \left[1-2 \cos [c+d x] \#1+\#1^2\right]+2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]\right. \\ \left. \#1^2-\frac{1}{2} \log \left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2\right] \Big/ \left(b-4 \frac{1}{2} a \#1-2 b \#1^2+b \#1^4\right) \&]$$

Problem 186: Result is not expressed in closed-form.

$$\int \frac{\csc [c+d x]}{a+b \sin [c+d x]^3} dx$$

Optimal (type 3, 264 leaves, 14 steps) :

$$-\frac{2 b^{1/3} \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3 a \sqrt{a^{2/3}-b^{2/3}} d}-\frac{\operatorname{ArcTanh}[\cos [c+d x]]}{a d}+$$

$$\frac{2 b^{1/3} \operatorname{ArcTanh}\left[\frac{b^{1/3}-(-1)^{1/3} a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{-(-1)^{2/3} a^{2/3}+b^{2/3}}}\right]}{3 a \sqrt{-(-1)^{2/3} a^{2/3}+b^{2/3}} d}+\frac{2 b^{1/3} \operatorname{ArcTanh}\left[\frac{b^{1/3}+(-1)^{2/3} a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{(-1)^{1/3} a^{2/3}+b^{2/3}}}\right]}{3 a \sqrt{(-1)^{1/3} a^{2/3}+b^{2/3}} d}$$

Result (type 7, 264 leaves) :

$$-\frac{1}{6 a d}\left(6 \log [\cos \left[\frac{1}{2} (c+d x)\right]]-6 \log [\sin \left[\frac{1}{2} (c+d x)\right]]+\right. \\ \left. \frac{1}{2} b \operatorname{RootSum}\left[-\frac{1}{2} b+3 \frac{1}{2} b \#1^2+8 a \#1^3-3 \frac{1}{2} b \#1^4+\frac{1}{2} b \#1^6 \&, \right. \right. \\ \left. \left. 2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]-\frac{1}{2} \log \left[1-2 \cos [c+d x] \#1+\#1^2\right]-4 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right]\right.\right. \\ \left. \left. \#1^2+2 \frac{1}{2} \log \left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^2+2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^4-\right. \right. \\ \left. \left. \frac{1}{2} \log \left[1-2 \cos [c+d x] \#1+\#1^2\right] \#1^4\right] \Big/ \left(b \#1-4 \frac{1}{2} a \#1^2-2 b \#1^3+b \#1^5\right) \& \right)$$

Problem 187: Result is not expressed in closed-form.

$$\int \frac{\csc [c+d x]^3}{a+b \sin [c+d x]^3} dx$$

Optimal (type 3, 287 leaves, 15 steps):

$$\begin{aligned}
 & -\frac{2 b \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3 a^{5/3} \sqrt{a^{2/3}-b^{2/3}} d} - \frac{2 b \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3}+a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}}}\right]}{3 a^{5/3} \sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}} d} + \\
 & \frac{2 b \operatorname{ArcTan}\left[\frac{(-1)^{1/3} \left(b^{1/3}+(-1)^{2/3} a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]\right)}{\sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}}}\right]}{3 a^{5/3} \sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}} d} - \frac{\operatorname{ArcTanh}[\cos[c+d x]]}{2 a d} - \frac{\cot[c+d x] \csc[c+d x]}{2 a d}
 \end{aligned}$$

Result (type 7, 181 leaves):

$$\begin{aligned}
 & \frac{1}{24 a d} \left(16 \operatorname{RootSum}\left[-b+3 b \#1^2-8 \operatorname{Root}\left[a \#1^3-3 b \#1^4+b \#1^6\right]\&, \right. \right. \\
 & \left. \left. \frac{2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x]-\#1}\right] \#1-\operatorname{Log}\left[1-2 \cos[c+d x] \#1+\#1^2\right] \#1}{b-4 \operatorname{Root}\left[a \#1-2 b \#1^2+b \#1^4\right]\&}\right] - \\
 & 3 \left(\csc\left[\frac{1}{2} (c+d x)\right]^2+4 \operatorname{Log}[\cos[\frac{1}{2} (c+d x)]]-4 \operatorname{Log}[\sin[\frac{1}{2} (c+d x)]]-\sec\left[\frac{1}{2} (c+d x)\right]^2 \right)
 \end{aligned}$$

Problem 188: Result is not expressed in closed-form.

$$\int \frac{\csc[c+d x]^5}{a+b \sin[c+d x]^3} dx$$

Optimal (type 3, 344 leaves, 18 steps):

$$\begin{aligned}
 & \frac{2 (-1)^{2/3} b^{5/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3}-a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}}}\right]}{3 a^{7/3} \sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}} d} - \frac{2 b^{5/3} \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3 a^{7/3} \sqrt{a^{2/3}-b^{2/3}} d} + \\
 & \frac{2 (-1)^{1/3} b^{5/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3}+a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}}}\right]}{3 a^{7/3} \sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}} d} - \frac{3 \operatorname{ArcTanh}[\cos[c+d x]]}{8 a d} + \\
 & \frac{b \cot[c+d x]}{a^2 d} - \frac{3 \cot[c+d x] \csc[c+d x]}{8 a d} - \frac{\cot[c+d x] \csc[c+d x]^3}{4 a d}
 \end{aligned}$$

Result (type 7, 290 leaves):

$$\frac{1}{192 a^2 d} \left(-64 b^2 \operatorname{RootSum} \left[-b + 3 b \#1^2 - 8 i a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \left(-2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] + i \operatorname{Log} \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] + 2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^2 - i \operatorname{Log} \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^2 \right) / \left(b - 4 i a \#1 - 2 b \#1^2 + b \#1^4 \right) \& \right] + 3 \left(32 b \operatorname{Cot} \left[\frac{1}{2} (c+d x) \right] - 6 a \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^2 - a \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right]^4 - 24 a \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+d x) \right] \right] + 24 a \operatorname{Log} \left[\sin \left[\frac{1}{2} (c+d x) \right] \right] + 6 a \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^2 + a \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^4 - 32 b \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] \right) \right)$$

Problem 189: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^6}{a + b \sin[c+d x]^3} dx$$

Optimal (type 3, 293 leaves, 15 steps):

$$-\frac{a x}{b^2} + \frac{2 a^{4/3} \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^2 d} + \frac{2 a^{4/3} \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} b^2 d} - \frac{2 a^{4/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/3} \left(b^{1/3} + (-1)^{2/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right] \right)}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} b^2 d} - \frac{\cos[c+d x]}{b d} + \frac{\cos[c+d x]^3}{3 b d}$$

Result (type 7, 164 leaves):

$$-\frac{1}{12 b^2 d} \left(12 a c + 12 a d x + 9 b \cos[c+d x] - b \cos[3(c+d x)] + 8 i a^2 \operatorname{RootSum} \left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \frac{2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1 - i \operatorname{Log} \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1}{b - 4 i a \#1 - 2 b \#1^2 + b \#1^4} \& \right] \right)$$

Problem 190: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^4}{a + b \sin[c+d x]^3} dx$$

Optimal (type 3, 281 leaves, 14 steps):

$$\begin{aligned}
& - \frac{2 (-1)^{2/3} a^{2/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} b^{4/3} d} + \frac{2 a^{2/3} \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^{4/3} d} - \\
& \frac{2 (-1)^{1/3} a^{2/3} \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} b^{4/3} d} - \frac{\cos [c + d x]}{b d}
\end{aligned}$$

Result (type 7, 186 leaves):

$$\begin{aligned}
& \frac{1}{3 b d} \left(-3 \cos [c + d x] + a \operatorname{RootSum} \left[-\frac{1}{2} b + 3 \frac{1}{2} b \#1^2 + 8 a \#1^3 - 3 \frac{1}{2} b \#1^4 + \frac{1}{2} b \#1^6 &, \right. \right. \\
& \left. \left. -2 \operatorname{ArcTan} \left[\frac{\sin [c + d x]}{\cos [c + d x] - \#1} \right] + \frac{1}{2} \log \left[1 - 2 \cos [c + d x] \#1 + \#1^2 \right] + 2 \operatorname{ArcTan} \left[\frac{\sin [c + d x]}{\cos [c + d x] - \#1} \right] \right. \right. \\
& \left. \left. \#1^2 - \frac{1}{2} \log \left[1 - 2 \cos [c + d x] \#1 + \#1^2 \right] \#1^2 \right] \Big/ (b - 4 \frac{1}{2} a \#1 - 2 b \#1^2 + b \#1^4) & \right)
\end{aligned}$$

Problem 191: Result is not expressed in closed-form.

$$\int \frac{\sin [c + d x]^2}{a + b \sin [c + d x]^3} dx$$

Optimal (type 3, 240 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^{2/3} d} - \\
& \frac{2 \operatorname{ArcTanh} \left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{-(-1)^{2/3} a^{2/3}, b^{2/3}}} \right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} b^{2/3} d} - \frac{2 \operatorname{ArcTanh} \left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{(-1)^{1/3} a^{2/3}, b^{2/3}}} \right]}{3 \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} b^{2/3} d}
\end{aligned}$$

Result (type 7, 231 leaves):

$$\begin{aligned}
& \frac{1}{6 d} \frac{1}{2} \operatorname{RootSum} \left[-\frac{1}{2} b + 3 \frac{1}{2} b \#1^2 + 8 a \#1^3 - 3 \frac{1}{2} b \#1^4 + \frac{1}{2} b \#1^6 &, \right. \\
& \left. \left(2 \operatorname{ArcTan} \left[\frac{\sin [c + d x]}{\cos [c + d x] - \#1} \right] - \frac{1}{2} \log \left[1 - 2 \cos [c + d x] \#1 + \#1^2 \right] - 4 \operatorname{ArcTan} \left[\frac{\sin [c + d x]}{\cos [c + d x] - \#1} \right] \right. \right. \\
& \left. \left. \#1^2 + 2 \frac{1}{2} \log \left[1 - 2 \cos [c + d x] \#1 + \#1^2 \right] \#1^2 + 2 \operatorname{ArcTan} \left[\frac{\sin [c + d x]}{\cos [c + d x] - \#1} \right] \#1^4 - \right. \right. \\
& \left. \left. \frac{1}{2} \log \left[1 - 2 \cos [c + d x] \#1 + \#1^2 \right] \#1^4 \right] \Big/ (b \#1 - 4 \frac{1}{2} a \#1^2 - 2 b \#1^3 + b \#1^5) & \right)
\end{aligned}$$

Problem 192: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \sin [c + d x]^3} dx$$

Optimal (type 3, 245 leaves, 11 steps):

$$\begin{aligned} & \frac{2 \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3}-b^{2/3}} d} + \\ & \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}} d} - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/3} \left(b^{1/3}+(-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{\sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}} d} \end{aligned}$$

Result (type 7, 126 leaves):

$$\begin{aligned} & -\frac{1}{3 d} 2 \operatorname{RootSum}\left[-i b+3 i b \# 1^2+8 a \# 1^3-3 i b \# 1^4+i b \# 1^6 \&, \right. \\ & \left. \frac{2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\# 1}\right] \# 1-i \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \# 1+\# 1^2\right] \# 1}{b-4 i a \# 1-2 b \# 1^2+b \# 1^4} \&\right] \end{aligned}$$

Problem 193: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Csc}[c+d x]^2}{a+b \operatorname{Sin}[c+d x]^3} d x$$

Optimal (type 3, 281 leaves, 15 steps):

$$\begin{aligned} & -\frac{2 (-1)^{2/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3}-a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}}}\right]}{3 a^{4/3} \sqrt{a^{2/3}-(-1)^{2/3} b^{2/3}} d} + \frac{2 b^{2/3} \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3 a^{4/3} \sqrt{a^{2/3}-b^{2/3}} d} - \\ & \frac{2 (-1)^{1/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3}+a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}}}\right]}{3 a^{4/3} \sqrt{a^{2/3}+(-1)^{1/3} b^{2/3}} d} - \frac{\operatorname{Cot}[c+d x]}{a d} \end{aligned}$$

Result (type 7, 196 leaves):

$$\begin{aligned} & \frac{1}{6 a d} \left(-3 \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right] + 2 b \operatorname{RootSum}\left[-b+3 b \# 1^2-8 i a \# 1^3-3 b \# 1^4+b \# 1^6 \&, \right. \right. \\ & \left. \left. \left(-2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\# 1}\right] + i \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \# 1+\# 1^2\right] + \right. \right. \right. \\ & \left. \left. \left. 2 \operatorname{ArcTan}\left[\frac{\operatorname{Sin}[c+d x]}{\operatorname{Cos}[c+d x]-\# 1}\right] \# 1^2 - i \operatorname{Log}\left[1-2 \operatorname{Cos}[c+d x] \# 1+\# 1^2\right] \# 1^2 \right) \right) / \\ & \left. \left. \left. \left(b-4 i a \# 1-2 b \# 1^2+b \# 1^4 \right) \& \right] + 3 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] \right) \right) \end{aligned}$$

Problem 194: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Csc}[c+d x]^4}{a+b \operatorname{Sin}[c+d x]^3} d x$$

Optimal (type 3, 296 leaves, 16 steps):

$$\begin{aligned} & \frac{2 b^{4/3} \operatorname{ArcTan}\left[\frac{b^{1/3}+a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3}-b^{2/3}}}\right]}{3 a^2 \sqrt{a^{2/3}-b^{2/3}} d} + \\ & \frac{b \operatorname{ArcTanh}[\cos[c+d x]]}{a^2 d} - \frac{2 b^{4/3} \operatorname{ArcTanh}\left[\frac{b^{1/3}-(-1)^{1/3} a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{-(-1)^{2/3} a^{2/3}+b^{2/3}}}\right]}{3 a^2 \sqrt{-(-1)^{2/3} a^{2/3}+b^{2/3}} d} - \\ & \frac{2 b^{4/3} \operatorname{ArcTanh}\left[\frac{b^{1/3}+(-1)^{2/3} a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{(-1)^{1/3} a^{2/3}+b^{2/3}}}\right]}{3 a^2 \sqrt{(-1)^{1/3} a^{2/3}+b^{2/3}} d} - \frac{\cot[c+d x]}{a d} - \frac{\cot[c+d x]^3}{3 a d} \end{aligned}$$

Result (type 7, 333 leaves):

$$\begin{aligned} & \frac{1}{24 a^2 d} \left(-8 a \cot\left[\frac{1}{2} (c+d x)\right] + 24 b \log[\cos\left[\frac{1}{2} (c+d x)\right]] - \right. \\ & 24 b \log[\sin\left[\frac{1}{2} (c+d x)\right]] + 4 i b^2 \operatorname{RootSum}\left[-b+3 b \#1^2-8 i a \#1^3-3 b \#1^4+b \#1^6 \&, \right. \\ & \left(2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x]-\#1}\right] - i \log[1-2 \cos[c+d x] \#1+\#1^2] - 4 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x]-\#1}\right] \right. \\ & \left. \#1^2+2 i \log[1-2 \cos[c+d x] \#1+\#1^2] \#1^2+2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x]-\#1}\right] \#1^4- \right. \\ & \left. i \log[1-2 \cos[c+d x] \#1+\#1^2] \#1^4 \right) / \left(b \#1-4 i a \#1^2-2 b \#1^3+b \#1^5 \right) \&] + \\ & 8 a \csc[c+d x]^3 \sin\left[\frac{1}{2} (c+d x)\right]^4 - \frac{1}{2} a \csc\left[\frac{1}{2} (c+d x)\right]^4 \sin[c+d x] + \\ & \left. 8 a \tan\left[\frac{1}{2} (c+d x)\right] \right) \end{aligned}$$

Problem 195: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^9}{a-b \sin[c+d x]^4} dx$$

Optimal (type 3, 177 leaves, 6 steps):

$$\begin{aligned} & -\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}-\sqrt{b}} b^{9/4} d} - \frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}+\sqrt{b}} b^{9/4} d} + \\ & \frac{(a+b) \cos[c+d x]}{b^2 d} - \frac{2 \cos[c+d x]^3}{3 b d} + \frac{\cos[c+d x]^5}{5 b d} \end{aligned}$$

Result (type 7, 228 leaves):

$$\frac{1}{120 b^2 d} \left(\cos[c + dx] (120 a + 89 b - 28 b \cos[2(c + dx)] + 3 b \cos[4(c + dx)]) + 60 i a^2 \operatorname{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \left(-2 \operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#1}\right] \#1 + i \operatorname{Log}[1 - 2 \cos[c + dx] \#1 + \#1^2] \#1 + 2 \operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#1}\right] \#1^3 - i \operatorname{Log}[1 - 2 \cos[c + dx] \#1 + \#1^2] \#1^3 \right) / (-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6) \&] \right)$$

Problem 196: Result is not expressed in closed-form.

$$\int \frac{\sin[c + dx]^7}{a - b \sin[c + dx]^4} dx$$

Optimal (type 3, 148 leaves, 6 steps) :

$$-\frac{a \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}-\sqrt{b}} b^{7/4} d} + \frac{a \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}+\sqrt{b}} b^{7/4} d} + \frac{\cos[c+d x]}{b d} - \frac{\cos[c+d x]^3}{3 b d}$$

Result (type 7, 310 leaves) :

$$\frac{1}{24 b d} \left(18 \cos[c + dx] - 2 \cos[3(c + dx)] - 3 i a \operatorname{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \left(-2 \operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#1}\right] + i \operatorname{Log}[1 - 2 \cos[c + dx] \#1 + \#1^2] + 6 \operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#1}\right] \#1^2 - 3 i \operatorname{Log}[1 - 2 \cos[c + dx] \#1 + \#1^2] \#1^2 - 6 \operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#1}\right] \#1^4 + 3 i \operatorname{Log}[1 - 2 \cos[c + dx] \#1 + \#1^2] \#1^4 + 2 \operatorname{ArcTan}\left[\frac{\sin[c + dx]}{\cos[c + dx] - \#1}\right] \#1^6 - i \operatorname{Log}[1 - 2 \cos[c + dx] \#1 + \#1^2] \#1^6 \right) / (-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7) \&] \right)$$

Problem 197: Result is not expressed in closed-form.

$$\int \frac{\sin[c + dx]^5}{a - b \sin[c + dx]^4} dx$$

Optimal (type 3, 138 leaves, 6 steps) :

$$-\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}-\sqrt{b}} b^{5/4} d} - \frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}+\sqrt{b}} b^{5/4} d} + \frac{\cos[c+d x]}{b d}$$

Result (type 7, 198 leaves) :

$$\begin{aligned} & \frac{1}{2 b d} \left(2 \cos[c + d x] + \right. \\ & \quad \pm a \operatorname{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \left(-2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1 + \right. \\ & \quad \pm \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1 + 2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^3 - \\ & \quad \left. \pm \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^3 \right) \Big/ (-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6) \& \Big] \end{aligned}$$

Problem 198: Result is not expressed in closed-form.

$$\int \frac{\sin[c + d x]^3}{a - b \sin[c + d x]^4} dx$$

Optimal (type 3, 115 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}-\sqrt{b}} b^{3/4} d} + \frac{\operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}+\sqrt{b}} b^{3/4} d}$$

Result (type 7, 285 leaves):

$$\begin{aligned} & -\frac{1}{8 d} \pm \operatorname{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\ & \quad \left(-2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] + \pm \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] + 6 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \right. \\ & \quad \#1^2 - 3 \pm \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^2 - 6 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 + \\ & \quad 3 \pm \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^4 + 2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^6 - \\ & \quad \left. \pm \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^6 \right) \Big/ (-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7) \& \Big] \end{aligned}$$

Problem 199: Result is not expressed in closed-form.

$$\int \frac{\sin[c + d x]}{a - b \sin[c + d x]^4} dx$$

Optimal (type 3, 125 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 \sqrt{a} \sqrt{\sqrt{a}-\sqrt{b}} b^{1/4} d} - \frac{\operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 \sqrt{a} \sqrt{\sqrt{a}+\sqrt{b}} b^{1/4} d}$$

Result (type 7, 183 leaves):

$$\frac{1}{2 d} \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \\ \left(-2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1 + i \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1 + \right. \\ \left. 2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^3 - i \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1^3 \right) / \\ (-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6) \&]$$

Problem 200: Result is not expressed in closed-form.

$$\int \frac{\csc[c+d x]}{a - b \sin[c+d x]^4} dx$$

Optimal (type 3, 136 leaves, 7 steps):

$$-\frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a \sqrt{\sqrt{a}-\sqrt{b}} d} - \frac{\operatorname{ArcTanh}[\cos[c+d x]]}{a d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a \sqrt{\sqrt{a}+\sqrt{b}} d}$$

Result (type 7, 318 leaves):

$$-\frac{1}{8 a d} \left(8 \operatorname{Log}[\cos[\frac{1}{2} (c+d x)]] - 8 \operatorname{Log}[\sin[\frac{1}{2} (c+d x)]] + \right. \\ \left. i b \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \right. \\ \left. \left(-2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + i \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] + \right. \right. \\ \left. \left. 6 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 - 3 i \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1^2 - \right. \right. \\ \left. \left. 6 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 + 3 i \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1^4 + \right. \right. \\ \left. \left. 2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - i \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1^6 \right) \right) / \\ (-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7) \& \Big)$$

Problem 201: Result is not expressed in closed-form.

$$\int \frac{\csc[c+d x]^3}{a - b \sin[c+d x]^4} dx$$

Optimal (type 3, 184 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{b^{3/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a^{3/2} \sqrt{\sqrt{a}-\sqrt{b}} d}-\frac{\operatorname{ArcTanh}[\cos [c+d x]]}{2 a d}- \\
 & \frac{b^{3/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a^{3/2} \sqrt{\sqrt{a}+\sqrt{b}} d}-\frac{1}{4 a d (1-\cos [c+d x])}+\frac{1}{4 a d (1+\cos [c+d x])}
 \end{aligned}$$

Result (type 7, 242 leaves) :

$$\begin{aligned}
 & \frac{1}{8 a d}\left(-\operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2-4 \operatorname{Log}[\cos [\frac{1}{2} (c+d x)]]+\right. \\
 & 4 \operatorname{Log}[\sin [\frac{1}{2} (c+d x)]]+4 \text{Integrate}[b \operatorname{RootSum}[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \\
 & \left.\left(-2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1+\text{Integrate}[\log [1-2 \cos [c+d x] \#1+\#1^2] \#1+\right. \\
 & \left.2 \operatorname{ArcTan}\left[\frac{\sin [c+d x]}{\cos [c+d x]-\#1}\right] \#1^3-\text{Integrate}[\log [1-2 \cos [c+d x] \#1+\#1^2] \#1^3\right]\right) \\
 & \left.\left(-b-8 a \#1^2+3 b \#1^2-3 b \#1^4+b \#1^6\right) \&\right]+\operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2\right)
 \end{aligned}$$

Problem 202: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Csc} [c+d x]^5}{a-b \sin [c+d x]^4} dx$$

Optimal (type 3, 229 leaves, 7 steps) :

$$\begin{aligned}
 & -\frac{b^{5/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a^2 \sqrt{\sqrt{a}-\sqrt{b}} d}-\frac{(3 a+8 b) \operatorname{ArcTanh}[\cos [c+d x]]}{8 a^2 d}+ \\
 & \frac{b^{5/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a^2 \sqrt{\sqrt{a}+\sqrt{b}} d}-\frac{1}{16 a d (1-\cos [c+d x])^2}- \\
 & \frac{3}{16 a d (1-\cos [c+d x])}+\frac{1}{16 a d (1+\cos [c+d x])^2}+\frac{3}{16 a d (1+\cos [c+d x])}
 \end{aligned}$$

Result (type 7, 409 leaves) :

$$\begin{aligned}
& \frac{1}{64 a^2 d} \left(-6 a \csc \left[\frac{1}{2} (c + d x) \right]^2 - a \csc \left[\frac{1}{2} (c + d x) \right]^4 - \right. \\
& 24 a \log [\cos \left[\frac{1}{2} (c + d x) \right]] - 64 b \log [\cos \left[\frac{1}{2} (c + d x) \right]] + 24 a \log [\sin \left[\frac{1}{2} (c + d x) \right]] + \\
& 64 b \log [\sin \left[\frac{1}{2} (c + d x) \right]] - 8 i b^2 \text{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\
& \left(-2 \arctan \left[\frac{\sin [c + d x]}{\cos [c + d x] - \#1} \right] + i \log [1 - 2 \cos [c + d x] \#1 + \#1^2] + \right. \\
& 6 \arctan \left[\frac{\sin [c + d x]}{\cos [c + d x] - \#1} \right] \#1^2 - 3 i \log [1 - 2 \cos [c + d x] \#1 + \#1^2] \#1^2 - \\
& 6 \arctan \left[\frac{\sin [c + d x]}{\cos [c + d x] - \#1} \right] \#1^4 + 3 i \log [1 - 2 \cos [c + d x] \#1 + \#1^2] \#1^4 + \\
& \left. 2 \arctan \left[\frac{\sin [c + d x]}{\cos [c + d x] - \#1} \right] \#1^6 - i \log [1 - 2 \cos [c + d x] \#1 + \#1^2] \#1^6 \right) / \\
& (-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7) \&] + 6 a \sec \left[\frac{1}{2} (c + d x) \right]^2 + a \sec \left[\frac{1}{2} (c + d x) \right]^4
\end{aligned}$$

Problem 212: Result is not expressed in closed-form.

$$\int \frac{\sin [c + d x]^9}{(a - b \sin [c + d x]^4)^2} dx$$

Optimal (type 3, 236 leaves, 7 steps):

$$\begin{aligned}
& \frac{\sqrt{a} \left(5 \sqrt{a} - 6 \sqrt{b} \right) \arctan \left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}} \right]}{8 \left(\sqrt{a} - \sqrt{b} \right)^{3/2} b^{9/4} d} + \frac{\sqrt{a} \left(5 \sqrt{a} + 6 \sqrt{b} \right) \operatorname{Arctanh} \left[\frac{b^{1/4} \cos [c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}} \right]}{8 \left(\sqrt{a} + \sqrt{b} \right)^{3/2} b^{9/4} d} - \\
& \frac{\cos [c + d x]}{b^2 d} - \frac{a \cos [c + d x] (a + b - b \cos [c + d x]^2)}{4 (a - b) b^2 d (a - b + 2 b \cos [c + d x]^2 - b \cos [c + d x]^4)}
\end{aligned}$$

Result (type 7, 486 leaves):

$$\begin{aligned}
& -\frac{1}{32 b^2 d} \\
& \left(32 \cos[c+d x] + \frac{32 a \cos[c+d x] (2 a + b - b \cos[2(c+d x)])}{(a-b) (8 a - 3 b + 4 b \cos[2(c+d x)] - b \cos[4(c+d x)])} + \frac{1}{a-b} \operatorname{i} a \operatorname{RootSum}[\right. \\
& b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \\
& \left(-2 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + \operatorname{i} b \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] - \right. \\
& 40 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + 54 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + \\
& 20 \operatorname{i} a \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1^2 - 27 \operatorname{i} b \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1^2 + \\
& 40 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - 54 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - \\
& 20 \operatorname{i} a \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1^4 + 27 \operatorname{i} b \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1^4 + \\
& \left. 2 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - \operatorname{i} b \operatorname{Log}\left[1 - 2 \cos[c+d x] \#1 + \#1^2\right] \#1^6 \right) \& \left. \right)
\end{aligned}$$

Problem 213: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^7}{(a-b \sin[c+d x]^4)^2} dx$$

Optimal (type 3, 210 leaves, 5 steps) :

$$\begin{aligned}
& \frac{\left(3 \sqrt{a} - 4 \sqrt{b}\right) \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} - \sqrt{b}}}\right]}{8 \left(\sqrt{a} - \sqrt{b}\right)^{3/2} b^{7/4} d} - \frac{\left(3 \sqrt{a} + 4 \sqrt{b}\right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} + \sqrt{b}}}\right]}{8 \left(\sqrt{a} + \sqrt{b}\right)^{3/2} b^{7/4} d} - \\
& \frac{a \cos[c+d x] (2 - \cos[c+d x]^2)}{4 (a-b) b d (a-b+2 b \cos[c+d x]^2 - b \cos[c+d x]^4)}
\end{aligned}$$

Result (type 7, 565 leaves) :

$$\begin{aligned}
& \frac{1}{32 (a-b) b d} \left(\frac{16 a (-5 \cos[c+d x] + \cos[3 (c+d x)])}{8 a - 3 b + 4 b \cos[2 (c+d x)] - b \cos[4 (c+d x)]} - \right. \\
& \quad \left. \frac{1}{b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \right. \\
& \quad \left(6 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - 8 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - \right. \\
& \quad \left. 3 \#1 a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] + 4 \#1 b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] - \right. \\
& \quad \left. 10 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + 24 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + \right. \\
& \quad \left. 5 \#1 a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - 12 \#1 b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + \right. \\
& \quad \left. 10 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - 24 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - \right. \\
& \quad \left. 5 \#1 a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + 12 \#1 b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - \right. \\
& \quad \left. 6 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + 8 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + \right. \\
& \quad \left. 3 \#1 a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 - 4 \#1 b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) \& \left. \right)
\end{aligned}$$

Problem 214: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^5}{(a - b \sin[c+d x]^4)^2} dx$$

Optimal (type 3, 217 leaves, 5 steps):

$$\begin{aligned}
& \frac{(\sqrt{a} - 2 \sqrt{b}) \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} - \sqrt{b}}}\right]}{8 \sqrt{a} (\sqrt{a} - \sqrt{b})^{3/2} b^{5/4} d} + \frac{(\sqrt{a} + 2 \sqrt{b}) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} + \sqrt{b}}}\right]}{8 \sqrt{a} (\sqrt{a} + \sqrt{b})^{3/2} b^{5/4} d} - \\
& \frac{\cos[c+d x] (a + b - b \cos[c+d x]^2)}{4 (a-b) b d (a - b + 2 b \cos[c+d x]^2 - b \cos[c+d x]^4)}
\end{aligned}$$

Result (type 7, 469 leaves):

$$\begin{aligned}
& - \frac{1}{32 (a-b) b d} \left(\frac{32 \cos[c+d x] (2 a + b - b \cos[2 (c+d x)])}{8 a - 3 b + 4 b \cos[2 (c+d x)] - b \cos[4 (c+d x)]} + \right. \\
& \quad \pm \text{RootSum}[b - 4 b \pm 1^2 - 16 a \pm 1^4 + 6 b \pm 1^4 - 4 b \pm 1^6 + b \pm 1^8 \&, \\
& \quad \frac{1}{-b \pm 1 - 8 a \pm 1^3 + 3 b \pm 1^3 - 3 b \pm 1^5 + b \pm 1^7} \left(-2 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1}\right] + \right. \\
& \quad \pm b \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] - 8 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1}\right] \pm 1^2 + \\
& \quad 22 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1}\right] \pm 1^2 + 4 \pm a \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^2 - \\
& \quad 11 \pm b \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^2 + 8 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1}\right] \pm 1^4 - \\
& \quad 22 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1}\right] \pm 1^4 - 4 \pm a \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^4 + \\
& \quad 11 \pm b \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^4 + 2 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1}\right] \pm 1^6 - \\
& \quad \left. \pm b \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^6 \right) \& \left. \right)
\end{aligned}$$

Problem 215: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^3}{(a-b \sin[c+d x]^4)^2} dx$$

Optimal (type 3, 186 leaves, 5 steps) :

$$\begin{aligned}
& - \frac{\operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8 \sqrt{a} \left(\sqrt{a}-\sqrt{b}\right)^{3/2} b^{3/4} d} + \frac{\operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 \sqrt{a} \left(\sqrt{a}+\sqrt{b}\right)^{3/2} b^{3/4} d} - \\
& \frac{\cos[c+d x] (2 - \cos[c+d x]^2)}{4 (a-b) d (a-b+2 b \cos[c+d x]^2 - b \cos[c+d x]^4)}
\end{aligned}$$

Result (type 7, 345 leaves) :

$$\begin{aligned} & \frac{1}{32 (a - b) d} \left(\frac{16 (-5 \cos[c + d x] + \cos[3 (c + d x)])}{8 a - 3 b + 4 b \cos[2 (c + d x)] - b \cos[4 (c + d x)]} - \right. \\ & \quad \text{i RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\ & \quad \left(-2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] + \text{i Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] + \right. \\ & \quad 14 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^2 - 7 \text{i Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^2 - \\ & \quad 14 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 + 7 \text{i Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^4 + \\ & \quad \left. 2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^6 - \text{i Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^6 \right) / \\ & \quad \left(-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7 \right) \& \left. \right) \end{aligned}$$

Problem 216: Result is not expressed in closed-form.

$$\int \frac{\sin[c + d x]}{(a - b \sin[c + d x]^4)^2} dx$$

Optimal (type 3, 221 leaves, 5 steps):

$$\begin{aligned} & -\frac{\left(3 \sqrt{a} - 2 \sqrt{b}\right) \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8 a^{3/2} \left(\sqrt{a} - \sqrt{b}\right)^{3/2} b^{1/4} d} - \frac{\left(3 \sqrt{a} + 2 \sqrt{b}\right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 a^{3/2} \left(\sqrt{a} + \sqrt{b}\right)^{3/2} b^{1/4} d} - \\ & \frac{\cos[c + d x] (a + b - b \cos[c + d x]^2)}{4 a (a - b) d (a - b + 2 b \cos[c + d x]^2 - b \cos[c + d x]^4)} \end{aligned}$$

Result (type 7, 469 leaves):

$$\begin{aligned}
& - \frac{1}{32 a (a - b) d} \left(\frac{32 \cos[c + d x] (2 a + b - b \cos[2 (c + d x)])}{8 a - 3 b + 4 b \cos[2 (c + d x)] - b \cos[4 (c + d x)]} + \right. \\
& \quad \text{i RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\
& \quad \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(-2 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] + \right. \\
& \quad \text{i } b \log[1 - 2 \cos[c + d x] \#1 + \#1^2] + 24 a \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^2 - \\
& \quad 10 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^2 - 12 \text{i } a \log[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^2 + \\
& \quad 5 \text{i } b \log[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^2 - 24 a \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 + \\
& \quad 10 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 + 12 \text{i } a \log[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^4 - \\
& \quad 5 \text{i } b \log[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^4 + 2 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^6 - \\
& \quad \left. \text{i } b \log[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^6 \right) \& \left. \right)
\end{aligned}$$

Problem 217: Result is not expressed in closed-form.

$$\int \frac{\csc[c + d x]}{(a - b \sin[c + d x]^4)^2} dx$$

Optimal (type 3, 325 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c + d x]}{\sqrt{\sqrt{a} - \sqrt{b}}}\right]}{8 a^{3/2} (\sqrt{a} - \sqrt{b})^{3/2} d} - \frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c + d x]}{\sqrt{\sqrt{a} + \sqrt{b}}}\right]}{2 a^2 \sqrt{\sqrt{a} + \sqrt{b}} d} - \\
& \frac{\operatorname{ArcTanh}[\cos[c + d x]]}{a^2 d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c + d x]}{\sqrt{\sqrt{a} + \sqrt{b}}}\right]}{8 a^{3/2} (\sqrt{a} + \sqrt{b})^{3/2} d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c + d x]}{\sqrt{\sqrt{a} - \sqrt{b}}}\right]}{2 a^2 \sqrt{\sqrt{a} - \sqrt{b}} d} - \\
& \frac{b \cos[c + d x] (2 - \cos[c + d x]^2)}{4 a (a - b) d (a - b + 2 b \cos[c + d x]^2 - b \cos[c + d x]^4)}
\end{aligned}$$

Result (type 7, 600 leaves):

$$\frac{1}{32 a^2 d} \left(\frac{16 a b (-5 \cos[c+d x] + \cos[3 (c+d x)])}{(a-b) (8 a - 3 b + 4 b \cos[2 (c+d x)] - b \cos[4 (c+d x)])} - \right.$$

$$32 \log[\cos[\frac{1}{2} (c+d x)]] + 32 \log[\sin[\frac{1}{2} (c+d x)]] - \frac{1}{a-b} i b \text{RootSum}[$$

$$b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7}$$

$$\left(-10 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + 8 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + \right.$$

$$5 i a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] - 4 i b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] +$$

$$38 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 - 24 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 -$$

$$19 i a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + 12 i b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 -$$

$$38 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 + 24 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 +$$

$$19 i a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - 12 i b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 +$$

$$10 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - 8 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 -$$

$$\left. 5 i a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 + 4 i b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) \& \left. \right)$$

Problem 224: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^9}{(a-b \sin[c+d x]^4)^3} dx$$

Optimal (type 3, 315 leaves, 6 steps):

$$-\frac{\left(5 a - 14 \sqrt{a} \sqrt{b} + 12 b\right) \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64 \sqrt{a} \left(\sqrt{a}-\sqrt{b}\right)^{5/2} b^{9/4} d} - \frac{\left(5 a + 14 \sqrt{a} \sqrt{b} + 12 b\right) \text{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 \sqrt{a} \left(\sqrt{a}+\sqrt{b}\right)^{5/2} b^{9/4} d} +$$

$$\frac{a \cos[c+d x] (a+b-b \cos[c+d x]^2)}{8 (a-b) b^2 d (a-b+2 b \cos[c+d x]^2-b \cos[c+d x]^4)^2} +$$

$$\frac{\cos[c+d x] (9 a^2 - 11 a b - 10 b^2 - 2 (2 a - 5 b) b \cos[c+d x]^2)}{32 (a-b)^2 b^2 d (a-b+2 b \cos[c+d x]^2-b \cos[c+d x]^4)}$$

Result (type 7, 785 leaves):

$$\begin{aligned}
& \frac{1}{128 (a-b)^2 b^2 d} \left(- \left((32 \cos[c+d x] (-9 a^2 + 13 a b + 5 b^2 + (2 a - 5 b) b \cos[2 (c+d x)])) / \right. \right. \\
& \quad \left(8 a - 3 b + 4 b \cos[2 (c+d x)] - b \cos[4 (c+d x)] \right)) - \\
& \quad \frac{512 a (a-b) \cos[c+d x] (2 a + b - b \cos[2 (c+d x)])}{(-8 a + 3 b - 4 b \cos[2 (c+d x)] + b \cos[4 (c+d x)])^2} + \\
& \quad \frac{1}{\text{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7}} \\
& \quad \left(-4 a b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + 10 b^2 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + \right. \\
& \quad 2 \pm a b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] - 5 \pm b^2 \log[1 - 2 \cos[c+d x] \#1 + \#1^2] - \\
& \quad 20 a^2 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + 56 a b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 - \\
& \quad 78 b^2 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + 10 \pm a^2 \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - \\
& \quad 28 \pm a b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + 39 \pm b^2 \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + \\
& \quad 20 a^2 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - 56 a b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 + \\
& \quad 78 b^2 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - 10 \pm a^2 \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + \\
& \quad 28 \pm a b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - 39 \pm b^2 \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + \\
& \quad 4 a b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - 10 b^2 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - \\
& \quad \left. 2 \pm a b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 + 5 \pm b^2 \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) \& \left. \right)
\end{aligned}$$

Problem 225: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^7}{(a - b \sin[c+d x]^4)^3} dx$$

Optimal (type 3, 290 leaves, 6 steps):

$$\begin{aligned}
& \frac{3 (\sqrt{a} - 2 \sqrt{b}) \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} - \sqrt{b}}}\right]}{64 \sqrt{a} (\sqrt{a} - \sqrt{b})^{5/2} b^{7/4} d} - \frac{3 (\sqrt{a} + 2 \sqrt{b}) \text{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} + \sqrt{b}}}\right]}{64 \sqrt{a} (\sqrt{a} + \sqrt{b})^{5/2} b^{7/4} d} - \\
& \frac{a \cos[c+d x] (2 - \cos[c+d x]^2)}{8 (a-b) b d (a-b+2 b \cos[c+d x]^2 - b \cos[c+d x]^4)^2} + \\
& \frac{\cos[c+d x] (5 a - 17 b - 3 (a-3 b) \cos[c+d x]^2)}{32 (a-b)^2 b d (a-b+2 b \cos[c+d x]^2 - b \cos[c+d x]^4)}
\end{aligned}$$

Result (type 7, 630 leaves):

$$\begin{aligned}
& \frac{1}{256 (a-b)^2 b d} \left(-\frac{32 \cos[c+d x] (-7 a + 25 b + 3 (a-3 b) \cos[2 (c+d x)])}{8 a - 3 b + 4 b \cos[2 (c+d x)] - b \cos[4 (c+d x)]} + \right. \\
& \frac{512 a (a-b) (-5 \cos[c+d x] + \cos[3 (c+d x)])}{(-8 a + 3 b - 4 b \cos[2 (c+d x)] + b \cos[4 (c+d x)])^2} - \\
& 3 \text{ i RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\
& \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(2 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - \right. \\
& 6 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - \text{i} a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] + \\
& 3 \text{i} b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] - 6 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + \\
& 34 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + 3 \text{i} a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - \\
& 17 \text{i} b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + 6 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - \\
& 34 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - 3 \text{i} a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + \\
& 17 \text{i} b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - 2 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + \\
& 6 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + \text{i} a \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 - \\
& \left. 3 \text{i} b \log[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) \&] \Bigg)
\end{aligned}$$

Problem 226: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^5}{(a - b \sin[c+d x]^4)^3} dx$$

Optimal (type 3, 313 leaves, 6 steps):

$$\begin{aligned}
& \frac{(3 a - 10 \sqrt{a} \sqrt{b} + 4 b) \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} - \sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a} - \sqrt{b})^{5/2} b^{5/4} d} + \frac{(3 a + 10 \sqrt{a} \sqrt{b} + 4 b) \text{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} + \sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a} + \sqrt{b})^{5/2} b^{5/4} d} - \\
& \frac{\cos[c+d x] (a + b - b \cos[c+d x]^2)}{8 (a - b) b d (a - b + 2 b \cos[c+d x]^2 - b \cos[c+d x]^4)^2} + \\
& \frac{\cos[c+d x] (a^2 - 11 a b - 2 b^2 + 2 b (2 a + b) \cos[c+d x]^2)}{32 a (a - b)^2 b d (a - b + 2 b \cos[c+d x]^2 - b \cos[c+d x]^4)}
\end{aligned}$$

Result (type 7, 786 leaves):

$$\begin{aligned}
& \frac{1}{128 (a-b)^2 b d} \left(\frac{32 \cos[c+d x] (a^2 - 9 a b - b^2 + b (2 a + b) \cos[2 (c+d x)])}{a (8 a - 3 b + 4 b \cos[2 (c+d x)] - b \cos[4 (c+d x)])} - \right. \\
& \frac{512 (a-b) \cos[c+d x] (2 a + b - b \cos[2 (c+d x)])}{(-8 a + 3 b - 4 b \cos[2 (c+d x)] + b \cos[4 (c+d x)])^2} + \\
& \frac{1}{a} \operatorname{iRootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\
& \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(4 a b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + \right. \\
& 2 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - 2 \operatorname{i} a b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] - \\
& \operatorname{i} b^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] + 12 a^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 - \\
& 64 a b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + 10 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 - \\
& 6 \operatorname{i} a^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + 32 \operatorname{i} a b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - \\
& 5 \operatorname{i} b^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - 12 a^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 + \\
& 64 a b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - 10 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 + \\
& 6 \operatorname{i} a^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - 32 \operatorname{i} a b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + \\
& 5 \operatorname{i} b^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - 4 a b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - \\
& 2 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + 2 \operatorname{i} a b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 + \\
& \left. \operatorname{i} b^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) \&] \Bigg)
\end{aligned}$$

Problem 227: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]^3}{(a - b \sin[c+d x]^4)^3} dx$$

Optimal (type 3, 288 leaves, 6 steps):

$$\begin{aligned}
& -\frac{\left(5 \sqrt{a} - 2 \sqrt{b}\right) \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} - \sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a} - \sqrt{b})^{5/2} b^{3/4} d} + \frac{\left(5 \sqrt{a} + 2 \sqrt{b}\right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a} + \sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a} + \sqrt{b})^{5/2} b^{3/4} d} - \\
& \frac{\cos[c+d x] (2 - \cos[c+d x]^2)}{8 (a - b) d (a - b + 2 b \cos[c+d x]^2 - b \cos[c+d x]^4)^2} - \\
& \frac{\cos[c+d x] (11 a + b - (5 a + b) \cos[c+d x]^2)}{32 a (a - b)^2 d (a - b + 2 b \cos[c+d x]^2 - b \cos[c+d x]^4)}
\end{aligned}$$

Result (type 7, 631 leaves):

$$\begin{aligned}
& \frac{1}{256 (a-b)^2 d} \left(\frac{32 \cos[c+d x] (-17 a - b + (5 a + b) \cos[2(c+d x)])}{a (8 a - 3 b + 4 b \cos[2(c+d x)] - b \cos[4(c+d x)])} + \right. \\
& \frac{512 (a-b) (-5 \cos[c+d x] + \cos[3(c+d x)])}{(-8 a + 3 b - 4 b \cos[2(c+d x)] + b \cos[4(c+d x)])^2} + \\
& \frac{1}{a} \operatorname{iRootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\
& \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(10 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + \right. \\
& 2 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - 5 \operatorname{i} a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] - \\
& \operatorname{i} b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] - 94 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + \\
& 10 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + 47 \operatorname{i} a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - \\
& 5 \operatorname{i} b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + 94 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - \\
& 10 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - 47 \operatorname{i} a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + \\
& 5 \operatorname{i} b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - 10 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - \\
& 2 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + 5 \operatorname{i} a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 + \\
& \left. \operatorname{i} b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) \&] \Bigg)
\end{aligned}$$

Problem 228: Result is not expressed in closed-form.

$$\int \frac{\sin[c+d x]}{(a-b \sin[c+d x]^4)^3} dx$$

Optimal (type 3, 313 leaves, 6 steps):

$$\begin{aligned}
& -\frac{3 (7 a - 10 \sqrt{a} \sqrt{b} + 4 b) \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64 a^{5/2} (\sqrt{a} - \sqrt{b})^{5/2} b^{1/4} d} - \frac{3 (7 a + 10 \sqrt{a} \sqrt{b} + 4 b) \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 a^{5/2} (\sqrt{a} + \sqrt{b})^{5/2} b^{1/4} d} - \\
& \frac{\cos[c+d x] (a + b - b \cos[c+d x]^2)}{8 a (a - b) d (a - b + 2 b \cos[c+d x]^2 - b \cos[c+d x]^4)^2} - \\
& \frac{\cos[c+d x] ((7 a - 3 b) (a + 2 b) - 6 (2 a - b) b \cos[c+d x]^2)}{32 a^2 (a - b)^2 d (a - b + 2 b \cos[c+d x]^2 - b \cos[c+d x]^4)}
\end{aligned}$$

Result (type 7, 784 leaves):

$$\begin{aligned}
& \frac{1}{128 a^2 (a-b)^2 d} \left(-\frac{32 \cos[c+d x] (7 a^2 + 5 a b - 3 b^2 + 3 b (-2 a + b) \cos[2 (c+d x)])}{8 a - 3 b + 4 b \cos[2 (c+d x)] - b \cos[4 (c+d x)]} - \right. \\
& \frac{512 a (a-b) \cos[c+d x] (2 a + b - b \cos[2 (c+d x)])}{(-8 a + 3 b - 4 b \cos[2 (c+d x)] + b \cos[4 (c+d x)])^2} + \\
& 3 \operatorname{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\
& \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(4 a b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - \right. \\
& 2 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - 2 \operatorname{i} a b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] + \\
& \operatorname{i} b^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] - 28 a^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + \\
& 24 a b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 - 10 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + \\
& 14 \operatorname{i} a^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - 12 \operatorname{i} a b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + \\
& 5 \operatorname{i} b^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + 28 a^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - \\
& 24 a b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 + 10 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - \\
& 14 \operatorname{i} a^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + 12 \operatorname{i} a b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - \\
& 5 \operatorname{i} b^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - 4 a b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + \\
& 2 b^2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + 2 \operatorname{i} a b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 - \\
& \left. \operatorname{i} b^2 \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) \& \Big)
\end{aligned}$$

Problem 229: Result is not expressed in closed-form.

$$\int \frac{\csc[c+d x]}{(a - b \sin[c+d x]^4)^3} dx$$

Optimal (type 3, 617 leaves, 16 steps):

$$\begin{aligned}
& - \frac{\left(5 \sqrt{a} - 2 \sqrt{b}\right) b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64 a^{5/2} (\sqrt{a}-\sqrt{b})^{5/2} d} - \frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 a^{5/2} (\sqrt{a}-\sqrt{b})^{3/2} d} - \\
& \frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a^3 \sqrt{\sqrt{a}-\sqrt{b}} d} - \frac{\operatorname{ArcTanh}[\cos[c+d x]]}{a^3 d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 a^{5/2} (\sqrt{a}+\sqrt{b})^{3/2} d} + \\
& \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a^3 \sqrt{\sqrt{a}+\sqrt{b}} d} + \frac{\left(5 \sqrt{a} + 2 \sqrt{b}\right) b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \cos[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 a^{5/2} (\sqrt{a}+\sqrt{b})^{5/2} d} - \\
& \frac{b \cos[c+d x] (2 - \cos[c+d x]^2)}{8 a (a-b) d (a-b+2 b \cos[c+d x]^2 - b \cos[c+d x]^4)^2} - \\
& \frac{b \cos[c+d x] (2 - \cos[c+d x]^2)}{4 a^2 (a-b) d (a-b+2 b \cos[c+d x]^2 - b \cos[c+d x]^4)} - \\
& \frac{b \cos[c+d x] (11 a + b - (5 a + b) \cos[c+d x]^2)}{32 a^2 (a-b)^2 d (a-b+2 b \cos[c+d x]^2 - b \cos[c+d x]^4)}
\end{aligned}$$

Result (type 7, 920 leaves):

$$\begin{aligned}
& \frac{1}{256 a^3 d} \left(\frac{32 a b \cos[c + d x] (-41 a + 23 b + (13 a - 7 b) \cos[2(c + d x)])}{(a - b)^2 (8 a - 3 b + 4 b \cos[2(c + d x)] - b \cos[4(c + d x)])} + \right. \\
& \quad \frac{512 a^2 b (-5 \cos[c + d x] + \cos[3(c + d x)])}{(a - b) (-8 a + 3 b - 4 b \cos[2(c + d x)] + b \cos[4(c + d x)])^2} - \\
& \quad 256 \operatorname{Log}[\cos[\frac{1}{2}(c + d x)]] + 256 \operatorname{Log}[\sin[\frac{1}{2}(c + d x)]] - \frac{1}{(a - b)^2} i b \operatorname{RootSum}[\\
& \quad b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \\
& \quad \left(-90 a^2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] + 142 a b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] - \right. \\
& \quad 64 b^2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] + 45 i a^2 \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] - \\
& \quad 71 i a b \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] + 32 i b^2 \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] + \\
& \quad 398 a^2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^2 - 506 a b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^2 + \\
& \quad 192 b^2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^2 - 199 i a^2 \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^2 + \\
& \quad 253 i a b \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^2 - 96 i b^2 \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^2 - \\
& \quad 398 a^2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 + 506 a b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 - \\
& \quad 192 b^2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 + 199 i a^2 \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^4 - \\
& \quad 253 i a b \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^4 + 96 i b^2 \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^4 + \\
& \quad 90 a^2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^6 - 142 a b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^6 + \\
& \quad 64 b^2 \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^6 - 45 i a^2 \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^6 + 71 i a \\
& \quad b \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^6 - 32 i b^2 \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^6 \Big) \& \Big] \\
\end{aligned}$$

Problem 237: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{a + b \sin[x]^4} dx$$

Optimal (type 3, 487 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\left(\sqrt{a} + \sqrt{a+b}\right) \operatorname{ArcTan}\left[\frac{a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}-\sqrt{2} (a+b)^{3/4} \tan[x]}{a^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}}\right]}{2 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}} + \\
& \frac{\left(\sqrt{a} + \sqrt{a+b}\right) \operatorname{ArcTan}\left[\frac{a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}+\sqrt{2} (a+b)^{3/4} \tan[x]}{a^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}}\right]}{2 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b+\sqrt{a}} \sqrt{a+b}} + \\
& \left(\left(\sqrt{a}-\sqrt{a+b}\right) \log \left[\sqrt{a} (a+b)^{1/4}-\sqrt{2} a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}\right] \tan[x]+(a+b)^{3/4} \tan[x]^2\right) / \\
& \left(4 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}\right) - \\
& \left(\left(\sqrt{a}-\sqrt{a+b}\right) \log \left[\sqrt{a} (a+b)^{1/4}+\sqrt{2} a^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}\right] \tan[x]+(a+b)^{3/4} \tan[x]^2\right) / \\
& \left(4 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b-\sqrt{a}} \sqrt{a+b}\right)
\end{aligned}$$

Result (type 3, 148 leaves):

$$\begin{aligned}
& \frac{1}{2 a (a+b)} \left(\left(\sqrt{a}-\pm i \sqrt{b}\right) \sqrt{a+\pm i \sqrt{a} \sqrt{b}} \operatorname{ArcTan}\left[\frac{\sqrt{a}+\pm i \sqrt{a} \sqrt{b} \tan[x]}{\sqrt{a}}\right] - \right. \\
& \left. \left(\sqrt{a}+\pm i \sqrt{b}\right) \sqrt{-a+\pm i \sqrt{a} \sqrt{b}} \operatorname{ArcTanh}\left[\frac{\sqrt{-a}+\pm i \sqrt{a} \sqrt{b} \tan[x]}{\sqrt{a}}\right] \right)
\end{aligned}$$

Problem 238: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1 + \sin[x]^4} dx$$

Optimal (type 3, 309 leaves, 10 steps):

$$\begin{aligned}
& \frac{x}{2 \sqrt{-1+\sqrt{2}}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{-1+\sqrt{2}}-2 \sqrt{-1+\sqrt{2}} \cos [x]^2-\left(-2+\sqrt{2}\right) \cos [x] \sin [x]}{2+\sqrt{1+\sqrt{2}}+\left(-2+\sqrt{2}\right) \cos [x]^2-2 \sqrt{-1+\sqrt{2}} \cos [x] \sin [x]}\right]}{4 \sqrt{-1+\sqrt{2}}} - \\
& \frac{\operatorname{ArcTan}\left[\frac{\sqrt{-1+\sqrt{2}}-2 \sqrt{-1+\sqrt{2}} \cos [x]^2+\left(-2+\sqrt{2}\right) \cos [x] \sin [x]}{2+\sqrt{1+\sqrt{2}}+\left(-2+\sqrt{2}\right) \cos [x]^2+2 \sqrt{-1+\sqrt{2}} \cos [x] \sin [x]}\right]}{4 \sqrt{-1+\sqrt{2}}} - \\
& \frac{\frac{1}{8} \sqrt{-1+\sqrt{2}} \log \left[\sqrt{2}-2 \sqrt{-1+\sqrt{2}} \tan [x]+2 \tan [x]^2\right]+}{\frac{1}{8} \sqrt{-1+\sqrt{2}} \log \left[1+\sqrt{2 \left(-1+\sqrt{2}\right)} \tan [x]+\sqrt{2} \tan [x]^2\right]}
\end{aligned}$$

Result (type 3, 45 leaves) :

$$\frac{\text{ArcTan}\left[\sqrt{1-i} \tan[x]\right]}{2 \sqrt{1-i}} + \frac{\text{ArcTan}\left[\sqrt{1+i} \tan[x]\right]}{2 \sqrt{1+i}}$$

Problem 239: Result unnecessarily involves imaginary or complex numbers.

$$\int \sin[c+d x] \sqrt{a+b \sin[c+d x]^4} dx$$

Optimal (type 4, 477 leaves, 5 steps) :

$$\begin{aligned} & -\frac{\cos[c+d x] \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}}{3 d} + \\ & \frac{2 \sqrt{b} \cos[c+d x] \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}}{3 \sqrt{a+b} d \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)} - \\ & \left(2 b^{1/4} (a+b)^{3/4} \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)^2}} \right. \\ & \left. \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\ & \left(3 d \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}\right) + \\ & \left((a+b)^{3/4} \left(\sqrt{b}-\sqrt{a+b}\right) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)^2}} \right. \\ & \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\ & \left(3 b^{1/4} d \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}\right) \end{aligned}$$

Result (type 4, 542 leaves) :

$$\begin{aligned}
& - \frac{\cos[c + d x] \sqrt{8 a + 3 b - 4 b \cos[2(c + d x)] + b \cos[4(c + d x)]}}{6 \sqrt{2} d} + \\
& \frac{1}{3 d \sqrt{\sec[c + d x]^2} (1 + \tan[c + d x]^2)^{3/2} \sqrt{\frac{b \tan[c + d x]^4 + a (1 + \tan[c + d x]^2)^2}{(1 + \tan[c + d x]^2)^2}}} \\
& 2 \sec[c + d x] \left(a + 2 a \tan[c + d x]^2 + a \tan[c + d x]^4 + b \tan[c + d x]^4 - \right. \\
& \left(-\frac{i \sqrt{a - i \sqrt{a} \sqrt{b} + a \tan[c + d x]^2 + b \tan[c + d x]^2}}{\sqrt{a} \sqrt{b}} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{i \sqrt{a - i \sqrt{a} \sqrt{b} + a \tan[c + d x]^2 + b \tan[c + d x]^2}}{\sqrt{2}}\right], \frac{2 \sqrt{a}}{\sqrt{a} - i \sqrt{b}}] + \right. \\
& \left. \left(\sqrt{a} + i \sqrt{b} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{i \sqrt{a - i \sqrt{a} \sqrt{b} + a \tan[c + d x]^2 + b \tan[c + d x]^2}}{\sqrt{2}}\right], \frac{\left(-\frac{i \sqrt{a} + \sqrt{b}}{\sqrt{a} - i \sqrt{b}}\right) (1 + \tan[c + d x]^2)}{\sqrt{b}} \right. \\
& \left. \left. \sqrt{\frac{i \left(a - i \sqrt{a} \sqrt{b} + a \tan[c + d x]^2 + b \tan[c + d x]^2\right)}{\sqrt{a} \sqrt{b}}} \right) \right. \\
& \left. \left(-\frac{i \sqrt{b} \tan[c + d x]^2 + \sqrt{a} (1 + \tan[c + d x]^2)}{\sqrt{a} \sqrt{b}} \right) \right) / \\
& \left(\sqrt{-\frac{i \left(a + i \sqrt{a} \sqrt{b} + a \tan[c + d x]^2 + b \tan[c + d x]^2\right)}{\sqrt{a} \sqrt{b}}} \right)
\end{aligned}$$

Problem 240: Unable to integrate problem.

$$\int \csc[c + d x] \sqrt{a + b \sin[c + d x]^4} dx$$

Optimal (type 4, 521 leaves, 8 steps) :

$$\begin{aligned}
 & \frac{\sqrt{-a} \operatorname{ArcTan}\left[\frac{\sqrt{-a} \cos [c+d x]}{\sqrt{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}}\right]}{2 d} + \\
 & \frac{\sqrt{b} \cos [c+d x] \sqrt{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}}{\sqrt{a+b} d \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right)} - \\
 & \left(b^{1/4} (a+b)^{3/4} \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right)^2}}\right. \\
 & \left.\operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \cos [c+d x]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
 & \left(d \sqrt{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}\right) - \\
 & \left((a+b)^{1/4} \left(\sqrt{b}-\sqrt{a+b}\right)^2 \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos [c+d x]^2}{\sqrt{a+b}}\right)^2}}\right. \\
 & \left.\operatorname{EllipticPi}\left[\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4 \sqrt{b} \sqrt{a+b}}, 2 \operatorname{ArcTan}\left[\frac{b^{1/4} \cos [c+d x]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
 & \left(4 b^{1/4} d \sqrt{a+b-2 b \cos [c+d x]^2+b \cos [c+d x]^4}\right)
 \end{aligned}$$

Result (type 8, 25 leaves) :

$$\int \csc [c+d x] \sqrt{a+b \sin [c+d x]^4} \mathrm{d} x$$

Problem 241: Attempted integration timed out after 120 seconds.

$$\int \frac{\sin [c+d x]^5}{\sqrt{a+b \sin [c+d x]^4}} \mathrm{d} x$$

Optimal (type 4, 484 leaves, 5 steps) :

$$\begin{aligned}
& - \frac{\cos[c + dx] \sqrt{a + b - 2 b \cos[c + dx]^2 + b \cos[c + dx]^4}}{3 b d} + \\
& \frac{2 \cos[c + dx] \sqrt{a + b - 2 b \cos[c + dx]^2 + b \cos[c + dx]^4}}{3 \sqrt{b} \sqrt{a + b} d \left(1 + \frac{\sqrt{b} \cos[c + dx]^2}{\sqrt{a + b}}\right)} - \\
& \left(2 (a + b)^{3/4} \left(1 + \frac{\sqrt{b} \cos[c + dx]^2}{\sqrt{a + b}}\right) \sqrt{\frac{a + b - 2 b \cos[c + dx]^2 + b \cos[c + dx]^4}{(a + b) \left(1 + \frac{\sqrt{b} \cos[c + dx]^2}{\sqrt{a + b}}\right)^2}} \right. \\
& \left. \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c + dx]}{(a + b)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{\sqrt{b}}{\sqrt{a + b}}\right)\right]\right) / \\
& \left(3 b^{3/4} d \sqrt{a + b - 2 b \cos[c + dx]^2 + b \cos[c + dx]^4}\right) + \left((a + b)^{1/4} (a - 2 b + 2 \sqrt{b} \sqrt{a + b}) \right. \\
& \left. \left(1 + \frac{\sqrt{b} \cos[c + dx]^2}{\sqrt{a + b}}\right) \sqrt{\frac{a + b - 2 b \cos[c + dx]^2 + b \cos[c + dx]^4}{(a + b) \left(1 + \frac{\sqrt{b} \cos[c + dx]^2}{\sqrt{a + b}}\right)^2}} \right. \\
& \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c + dx]}{(a + b)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{\sqrt{b}}{\sqrt{a + b}}\right)\right]\right) / \\
& \left(6 b^{5/4} d \sqrt{a + b - 2 b \cos[c + dx]^2 + b \cos[c + dx]^4}\right)
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 242: Result more than twice size of optimal antiderivative.

$$\int \frac{\sin[c + dx]^3}{\sqrt{a + b \sin[c + dx]^4}} dx$$

Optimal (type 4, 431 leaves, 4 steps):

$$\begin{aligned}
& \frac{\cos[c+dx] \sqrt{a+b-2b \cos[c+dx]^2+b \cos[c+dx]^4}}{\sqrt{b} \sqrt{a+b} d \left(1+\frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right)} - \\
& \left((a+b)^{3/4} \left(1+\frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2b \cos[c+dx]^2+b \cos[c+dx]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \right. \\
& \left. \text{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
& \left(b^{3/4} d \sqrt{a+b-2b \cos[c+dx]^2+b \cos[c+dx]^4} \right) - \\
& \left((a+b)^{1/4} \left(\sqrt{b}-\sqrt{a+b}\right) \left(1+\frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2b \cos[c+dx]^2+b \cos[c+dx]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \right. \\
& \left. \text{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
& \left(2 b^{3/4} d \sqrt{a+b-2b \cos[c+dx]^2+b \cos[c+dx]^4} \right)
\end{aligned}$$

Result (type 4, 35489 leaves) : Display of huge result suppressed!

Problem 243: Result more than twice size of optimal antiderivative.

$$\int \frac{\sin[c+dx]}{\sqrt{a+b \sin[c+dx]^4}} dx$$

Optimal (type 4, 171 leaves, 2 steps) :

$$\begin{aligned}
& - \left((a+b)^{1/4} \left(1+\frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2b \cos[c+dx]^2+b \cos[c+dx]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos[c+dx]^2}{\sqrt{a+b}}\right)^2}} \right. \\
& \left. \text{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \cos[c+dx]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
& \left(2 b^{1/4} d \sqrt{a+b-2b \cos[c+dx]^2+b \cos[c+dx]^4} \right)
\end{aligned}$$

Result (type 4, 13300 leaves):

$$\begin{aligned}
 & - \left(8 \sqrt{2} \operatorname{EllipticF} [\operatorname{ArcSin}[\sqrt{\left((\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \right.} \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \\
 & \quad \left. \left. \left(-\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right. \\
 & \quad \left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
 & \quad \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right. \\
 & \quad \left. \left. \left(-\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right), \\
 & \left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \right. \right. \\
 & \quad \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
 & \quad \left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
 & \quad \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right) / \\
 & \left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
 & \quad \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
 & \quad \left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \right. \right. \\
 & \quad \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right)] \\
 & \left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
 & \quad \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right) \sin[c + d x] \\
 & \left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \right. \\
 & \quad \left. \sqrt{\left(\left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \right) \right. \\
 & \quad \left. \left. \left. \left(-\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \right. \\
 & \quad \left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
 & \quad \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] \right) \right. \\
 & \quad \left. \left. \left(-\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \\
 & \quad \left. \sqrt{\left(\left(\left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \right) \right. \\
 & \quad \left. \left. \left. \left. \left(\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \right) \right. \\
 & \quad \left. \left. \left. \left. \left(-\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \right. \\
 & \quad \left. \left. \left. \left. \left(-\operatorname{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]^2 \\
& \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \Big) \Big) \Big) \Big) / \\
& \sqrt{d \sqrt{8 a + 3 b - 4 b \cos[2 (c + d x)]} + b \cos[4 (c + d x)]} \\
& (-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \\
& \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right]) \\
& (\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \\
& \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]) \\
& \left(1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right)^2 \sqrt{\frac{16 b \tan\left[\frac{1}{2} (c + d x)\right]^4 + a \left(1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right)^4}{\left(1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right)^4}} \\
& \left(- \left(8 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right]\right) \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2\right)\right)\right) \right. \right. \right. \\
& \left. \left. \left. \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2\right)\right)\right], \right. \\
& \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right]\right) \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \right) / \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right]\right) \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \right) \\
& \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \sec\left[\frac{1}{2} (c + d x)\right]^2 \tan\left[\frac{1}{2} (c + d x)\right] \\
& \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2\right) \\
& \sqrt{\left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right]\right)\right)}
\end{aligned}$$

$$\begin{aligned}
 & \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \Bigg) \Bigg/ \\
 & \left((\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \right. \\
 & \quad \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right]) \right. \\
 & \quad \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \Bigg) \\
 & \sqrt{\left((\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \right. \\
 & \quad \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right]\right) \\
 & \quad (\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \\
 & \quad \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]) \\
 & \quad \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \\
 & \quad \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \Bigg) \Bigg) \\
 & \left((\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \right. \\
 & \quad \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right)^2 \left(-\text{Root}\left[a + 4 a \#1 + \right. \right. \\
 & \quad \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \Bigg) \Bigg) \\
 & \left((-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + 4 a \#1 + \right. \right. \\
 & \quad \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right]) \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + \right. \right. \\
 & \quad \left. \left. 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \right) \right. \\
 & \quad \left. \left(1 + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \sqrt{\frac{16 b \tan\left[\frac{1}{2} (c + d x)\right]^4 + a \left(1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right)^4}{\left(1 + \tan\left[\frac{1}{2} (c + d x)\right]^2\right)^4}} \right) \Bigg) + \\
 & \left(8 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left((\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \right. \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \left. \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \right. \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \left. \left. \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right. \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \left. \left. \left. \left((\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \right. \right. \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \left. \left. \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \left(-\text{Root}\left[\right. \right. \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \left. \left. \left. \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right] \right], \\
 & \quad \left((\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + \right. \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \left. \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right]\right) \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right]\right) \right. \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \left. \left. \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right] \right] \right] \right] \right]
 \end{aligned}$$

$$\begin{aligned}
& \frac{\#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4])}{((\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + \\
& (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3]) (\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \\
& \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4]))} \\
& (\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[\\
& a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4]) \sec[\frac{1}{2} (c + d x)]^2 \tan[\frac{1}{2} (c + d x)] \\
& \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \tan[\frac{1}{2} (c + d x)]^2 \right)^2 \\
& \sqrt{\left(\left((\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
& \left. \left. \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2]) \right. \right. \\
& \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3] + \tan[\frac{1}{2} (c + d x)]^2 \right) \right) / \\
& \left((\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
& \left. \left. \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3]) \right. \right. \\
& \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \tan[\frac{1}{2} (c + d x)]^2 \right) \right) \\
& \sqrt{\left(\left((\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + 4 a \#1 + \right. \right. \\
& (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2]) (\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + \\
& 4 a \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4]) \\
& \left. \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \tan[\frac{1}{2} (c + d x)]^2 \right) \right) / \\
& \left(-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] + \tan[\frac{1}{2} (c + d x)]^2 \right) \right) / \\
& \left((\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
& \left. \left. \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4])^2 \left(-\text{Root}[\right. \right. \\
& a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \tan[\frac{1}{2} (c + d x)]^2 \right)^2 \right) \right) / \\
& (-\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \text{Root}[\\
& a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2]) \\
& (\text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \\
& \text{Root}[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4]) \\
& \left(1 + \tan[\frac{1}{2} (c + d x)]^2 \right)^3 \sqrt{\frac{16 b \tan[\frac{1}{2} (c + d x)]^4 + a \left(1 + \tan[\frac{1}{2} (c + d x)]^2 \right)^4}{\left(1 + \tan[\frac{1}{2} (c + d x)]^2 \right)^4}} -
\end{aligned}$$

$$\begin{aligned}
& \left(2 \left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \\
& \quad \sqrt{\left(\left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \right. \\
& \quad \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \Big) / \\
& \quad \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] \right) \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \Big) \Big) / \\
& \quad \sqrt{\left(\left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \text{Root} \left[a + 4 a \#1 + \right. \right. \right. \\
& \quad \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + \right. \right. \\
& \quad \left. \left. 4 a \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \Big) \Big) / \\
& \quad \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right)^2 \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \Big) \Big) \\
& \quad \left(-\left(\left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[a + 4 a \#1 + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \right. \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \Big) \Big) / \\
& \quad \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \left(-\text{Root} \left[a + 4 a \#1 + \right. \right. \\
& \quad \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \Big) + \\
& \quad \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[a + 4 a \#1 + \right. \right. \right. \\
& \quad \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \Big) \Big) /
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \right. \\
& \left. \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) / \\
& \left(\left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \right. \right. \\
& \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \right) \\
& \left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \\
& \sqrt{\left(\left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \right. \right. \right. \\
& \left. \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \right. \right. \\
& \left. \left. \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) / \\
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \right. \\
& \left. \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) \\
& \sqrt{\frac{16 b \tan \left[\frac{1}{2} (c + d x) \right]^4 + a \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^4}{\left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^4}} \\
& \sqrt{\left(1 - \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] \right) \right. \right. \\
& \left. \left. \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) / \\
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] \right) \right. \\
& \left. \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) \\
& \sqrt{\left(1 - \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \right. \right. \\
& \left. \left. \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) / \\
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] \left(-\text{Root}\left[\right. \right. \\
& \quad \left. \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \Bigg) - \\
& \left(2 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right]\right) - \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. -\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right. \right. \right. \right. \right. \right) \Bigg) / \\
& \quad \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. -\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \right. \right. \right. \right. \right. \right), \\
& \quad \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[\right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right]\right) \right. \right. \right. \right. \right. \right. \\
& \quad \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \right. \right. \right. \right. \right. \right) / \\
& \quad \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3\right]\right) \right. \right. \right. \right. \right. \right. \\
& \quad \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] - \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \right. \right. \right. \right. \right. \right)] \\
& \quad \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \right. \right. \right. \right. \right. \right. \\
& \quad \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \\
& \quad \sqrt{\left(\left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right]\right) \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. 4 a \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right]\right) \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \\
& \quad \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4\right] + \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \Bigg) / \\
& \quad \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + 4 a \#1 + \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2\right]\right) \sec\left[\frac{1}{2} (c + d x)\right]^2 \tan\left[\frac{1}{2} (c + d x)\right] \right) \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] \right) \right. \\
& \quad \left. \left(- \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) - \\
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \text{Root} \left[a + 4 a \#1 + \right. \right. \right. \\
& \quad \left. \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \\
& \quad \left(- \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) / \\
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] \right) \left(- \text{Root} \left[\right. \right. \right. \\
& \quad \left. \left. \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) / \\
& \left(\left(- \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[\right. \right. \right. \\
& \quad \left. \left. \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) - \\
& \quad \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \\
& \quad \sqrt{\left(\left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \right) \right. \\
& \quad \left(\left(- \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) / \\
& \quad \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] \right) \right. \\
& \quad \left(\left(- \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) \\
& \quad \sqrt{\frac{16 b \tan \left[\frac{1}{2} (c + d x) \right]^4 + a \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^4}{\left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^4}} \Bigg) - \\
& \left(2 \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\left(\left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \right. \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \right) \right) \right. \right. \right. \\
& \quad \left(\left(- \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \left(-\text{Root} \left[\right. \right. \right. \\
& \quad \left. \left. \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \left. \right), \\
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[\right. \right. \right. \\
& \quad \left. \left. \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] \right) \\
& \quad \left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \\
& \quad \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \right) / \\
& \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \\
& \quad \sqrt{\left(\left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \right) / \\
& \quad \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 3 \right] \right) \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \left. \right) \\
& \quad \left(\left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \right. \right. \right. \\
& \quad \left. \left. \left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[a + 4 a \#1 + \right. \right. \right. \\
& \quad \left. \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \right. \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \left. \right) / \\
& \quad \left(\left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right)^2 \\
& \quad \left(-\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) - \\
& \quad \left(2 \left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \left. \text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \right. \\
& \quad \left(\text{Root} \left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[a + 4 a \#1 + \right. \right. \\
& \quad \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right]
\end{aligned}$$

$$\begin{aligned}
& \left(6 a + 16 b \right) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4]) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \\
& \left(-\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \\
& \left(-\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \Bigg) / \\
& \left(\left(\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
& \quad \left. \operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right)^2 \\
& \quad \left(-\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^3 \Bigg) + \\
& \left(\left(\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
& \quad \left. \operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \\
& \quad \left(\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \operatorname{Root} [a + 4 a \#1 + \right. \\
& \quad \left. \left. (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right. \\
& \quad \left(-\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \Bigg) / \\
& \left(\left(\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
& \quad \left. \operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right)^2 \left(-\operatorname{Root} [\right. \\
& \quad \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \Bigg) \Bigg) / \\
& \left(\left(-\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \operatorname{Root} [\right. \right. \\
& \quad \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \\
& \quad \left(\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \right. \\
& \quad \left. \operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \\
& \quad \sqrt{ \left(\left(\left(\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \right. \\
& \quad \left. \operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] \right) \\
& \quad \left(\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2] - \right. \\
& \quad \left. \operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right) \\
& \quad \left(-\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \Bigg) \\
& \quad \left(-\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \Bigg) / \\
& \left(\left(\operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1] - \right. \right. \\
& \quad \left. \operatorname{Root} [a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4] \right)^2
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] + \tan\left[\frac{1}{2} (c + d x) \right]^2 \right) \Bigg) \Bigg/ \\
& \left(\left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right)^2 \right. \\
& \quad \left. \left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] + \tan\left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \Bigg) \\
& \left(\left(32 b \sec\left[\frac{1}{2} (c + d x) \right]^2 \tan\left[\frac{1}{2} (c + d x) \right]^3 + 4 a \sec\left[\frac{1}{2} (c + d x) \right]^2 \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (c + d x) \right] \left(1 + \tan\left[\frac{1}{2} (c + d x) \right]^2 \right)^3 \right) \Bigg/ \left(1 + \tan\left[\frac{1}{2} (c + d x) \right]^2 \right)^4 - \right. \\
& \quad \left. \left(4 \sec\left[\frac{1}{2} (c + d x) \right]^2 \tan\left[\frac{1}{2} (c + d x) \right] \left(16 b \tan\left[\frac{1}{2} (c + d x) \right]^4 + \right. \right. \right. \\
& \quad \left. \left. \left. a \left(1 + \tan\left[\frac{1}{2} (c + d x) \right]^2 \right)^4 \right) \right) \Bigg/ \left(1 + \tan\left[\frac{1}{2} (c + d x) \right]^2 \right)^5 \right) \Bigg) \Bigg/ \\
& \left(\left(-\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[\right. \right. \right. \\
& \quad \left. \left. \left. a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] \right) \right. \\
& \quad \left. \left(\text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 2 \right] - \right. \right. \\
& \quad \left. \left. \text{Root}\left[a + 4 a \#1 + (6 a + 16 b) \#1^2 + 4 a \#1^3 + a \#1^4 \&, 4 \right] \right) \right. \\
& \quad \left. \left(1 + \tan\left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \left(\frac{16 b \tan\left[\frac{1}{2} (c + d x) \right]^4 + a \left(1 + \tan\left[\frac{1}{2} (c + d x) \right]^2 \right)^4}{\left(1 + \tan\left[\frac{1}{2} (c + d x) \right]^2 \right)^4} \right)^{3/2} \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

Problem 244: Unable to integrate problem.

$$\int \frac{\csc[c + d x]}{\sqrt{a + b \sin[c + d x]^4}} dx$$

Optimal (type 4, 469 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{\sqrt{-a} \cos[c+d x]}{\sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}}\right]}{2 \sqrt{-a} d} + \\
& \left(\frac{b^{1/4} (a+b)^{1/4} (\sqrt{b}-\sqrt{a+b}) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)}{(a+b) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)^2} \sqrt{\frac{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)^2}} \right. \\
& \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
& \left(2 a d \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}\right) - \\
& \left((a+b)^{1/4} (\sqrt{b}-\sqrt{a+b})^2 \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)^2}} \right. \\
& \left. \text{EllipticPi}\left[\frac{(\sqrt{b}+\sqrt{a+b})^2}{4 \sqrt{b} \sqrt{a+b}}, 2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
& \left(4 a b^{1/4} d \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}\right)
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\csc[c+d x]}{\sqrt{a+b \sin[c+d x]^4}} dx$$

Problem 245: Attempted integration timed out after 120 seconds.

$$\int \frac{\csc[c+d x]^3}{\sqrt{a+b \sin[c+d x]^4}} dx$$

Optimal (type 4, 776 leaves, 7 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{\sqrt{-a} \cos[c+d x]}{\sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}}\right]}{4 \sqrt{-a} d} \\
& - \frac{\sqrt{b} \cos[c+d x] \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}}{2 a \sqrt{a+b} d \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)} \\
& + \frac{\sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4} \cot[c+d x] \csc[c+d x]}{2 a d} \\
& \left(\frac{b^{1/4} (a+b)^{3/4} \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)}{\sqrt{(a+b) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)^2}} \right. \\
& \left. \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
& \left(2 a d \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}\right) - \\
& \left(\frac{b^{1/4} (a+b-\sqrt{b} \sqrt{a+b}) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)}{\sqrt{(a+b) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)^2}} \right. \\
& \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
& \left(2 a (a+b)^{1/4} d \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}\right) - \\
& \left((a+b)^{1/4} (\sqrt{b}-\sqrt{a+b})^2 \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right) \sqrt{\frac{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}{(a+b) \left(1+\frac{\sqrt{b} \cos[c+d x]^2}{\sqrt{a+b}}\right)^2}} \right. \\
& \left. \text{EllipticPi}\left[\frac{(\sqrt{b}+\sqrt{a+b})^2}{4 \sqrt{b} \sqrt{a+b}}, 2 \text{ArcTan}\left[\frac{b^{1/4} \cos[c+d x]}{(a+b)^{1/4}}\right], \frac{1}{2} \left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right]\right) / \\
& \left(8 a b^{1/4} d \sqrt{a+b-2 b \cos[c+d x]^2+b \cos[c+d x]^4}\right)
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 246: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin[c + dx]^2}{\sqrt{a + b \sin[c + dx]^4}} dx$$

Optimal (type 4, 499 leaves, 4 steps) :

$$\begin{aligned}
 & - \left(\left(\text{ArcTan} \left[\frac{\sqrt{b} \tan[c + dx]}{\sqrt{a + 2a \tan[c + dx]^2 + (a + b) \tan[c + dx]^4}} \right] \cos[c + dx]^2 \right. \right. \\
 & \quad \left. \left. \left. \sqrt{a + 2a \tan[c + dx]^2 + (a + b) \tan[c + dx]^4} \right) \middle/ \left(2\sqrt{b} d \sqrt{a + b \sin[c + dx]^4} \right) \right) - \\
 & \quad \left(a^{1/4} \left(\sqrt{a} + \sqrt{a + b} \right) \cos[c + dx]^2 \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{(a + b)^{1/4} \tan[c + dx]}{a^{1/4}} \right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a + b}} \right) \right] \right. \\
 & \quad \left. \left. \left(\sqrt{a} + \sqrt{a + b} \tan[c + dx]^2 \right) \sqrt{\frac{a + 2a \tan[c + dx]^2 + (a + b) \tan[c + dx]^4}{\left(\sqrt{a} + \sqrt{a + b} \tan[c + dx]^2 \right)^2}} \right) \middle/ \right. \\
 & \quad \left. \left. \left(2b (a + b)^{1/4} d \sqrt{a + b \sin[c + dx]^4} \right) + \left(\left(\sqrt{a} + \sqrt{a + b} \right)^2 \cos[c + dx]^2 \right. \right. \\
 & \quad \left. \left. \left. \text{EllipticPi} \left[-\frac{(\sqrt{a} - \sqrt{a + b})^2}{4\sqrt{a}\sqrt{a + b}}, 2 \text{ArcTan} \left[\frac{(a + b)^{1/4} \tan[c + dx]}{a^{1/4}} \right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a + b}} \right) \right] \right) \right. \\
 & \quad \left. \left. \left. \left(\sqrt{a} + \sqrt{a + b} \tan[c + dx]^2 \right) \sqrt{\frac{a + 2a \tan[c + dx]^2 + (a + b) \tan[c + dx]^4}{\left(\sqrt{a} + \sqrt{a + b} \tan[c + dx]^2 \right)^2}} \right) \middle/ \right. \\
 & \quad \left. \left. \left. \left(4a^{1/4}b(a + b)^{1/4}d\sqrt{a + b \sin[c + dx]^4} \right) \right. \right. \right)
 \end{aligned}$$

Result (type 4, 287 leaves) :

$$\begin{aligned}
& - \left(\left(2 \operatorname{Cos}[c + d x]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\sqrt{1 - \frac{\operatorname{i} \sqrt{b}}{\sqrt{a}}} \operatorname{Tan}[c + d x] \right], \frac{\sqrt{a} + \operatorname{i} \sqrt{b}}{\sqrt{a} - \operatorname{i} \sqrt{b}} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{EllipticPi}\left[\frac{\sqrt{a}}{\sqrt{a} - \operatorname{i} \sqrt{b}}, \operatorname{ArcSinh}\left[\sqrt{1 - \frac{\operatorname{i} \sqrt{b}}{\sqrt{a}}} \operatorname{Tan}[c + d x] \right], \frac{\sqrt{a} + \operatorname{i} \sqrt{b}}{\sqrt{a} - \operatorname{i} \sqrt{b}} \right] \right) \right. \\
& \quad \left. \left. \left. \sqrt{1 + \left(1 + \frac{\operatorname{i} \sqrt{b}}{\sqrt{a}} \right) \operatorname{Tan}[c + d x]^2} \sqrt{2 + \left(2 - \frac{2 \operatorname{i} \sqrt{b}}{\sqrt{a}} \right) \operatorname{Tan}[c + d x]^2} \right) / \right. \\
& \quad \left. \left. \left. \left(\sqrt{1 - \frac{\operatorname{i} \sqrt{b}}{\sqrt{a}}} d \sqrt{8 a + 3 b - 4 b \operatorname{Cos}[2(c + d x)] + b \operatorname{Cos}[4(c + d x)]} \right) \right) \right)
\end{aligned}$$

Problem 247: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b \operatorname{Sin}[c + d x]^4}} dx$$

Optimal (type 4, 162 leaves, 2 steps) :

$$\begin{aligned}
& \left(\operatorname{Cos}[c + d x]^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{(a + b)^{1/4} \operatorname{Tan}[c + d x]}{a^{1/4}} \right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a + b}} \right) \right] \right. \\
& \quad \left. \left(\sqrt{a} + \sqrt{a + b} \operatorname{Tan}[c + d x]^2 \right) \sqrt{\frac{a + 2 a \operatorname{Tan}[c + d x]^2 + (a + b) \operatorname{Tan}[c + d x]^4}{\left(\sqrt{a} + \sqrt{a + b} \operatorname{Tan}[c + d x]^2 \right)^2}} \right) / \\
& \quad \left(2 a^{1/4} (a + b)^{1/4} d \sqrt{a + b \operatorname{Sin}[c + d x]^4} \right)
\end{aligned}$$

Result (type 4, 195 leaves) :

$$\begin{aligned}
& - \left(\left(2 \operatorname{Cos}[c + d x]^2 \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\sqrt{1 - \frac{\operatorname{i} \sqrt{b}}{\sqrt{a}}} \operatorname{Tan}[c + d x] \right], \frac{\sqrt{a} + \operatorname{i} \sqrt{b}}{\sqrt{a} - \operatorname{i} \sqrt{b}} \right] \right. \right. \\
& \quad \left. \left. \left. \sqrt{1 + \left(1 + \frac{\operatorname{i} \sqrt{b}}{\sqrt{a}} \right) \operatorname{Tan}[c + d x]^2} \sqrt{2 + \left(2 - \frac{2 \operatorname{i} \sqrt{b}}{\sqrt{a}} \right) \operatorname{Tan}[c + d x]^2} \right) / \right. \\
& \quad \left. \left. \left. \left(\sqrt{1 - \frac{\operatorname{i} \sqrt{b}}{\sqrt{a}}} d \sqrt{8 a + 3 b - 4 b \operatorname{Cos}[2(c + d x)] + b \operatorname{Cos}[4(c + d x)]} \right) \right) \right)
\end{aligned}$$

Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[(c+d x)^2]}{\sqrt{a+b \operatorname{Sin}[(c+d x)^4]}} d x$$

Optimal (type 4, 493 leaves, 5 steps):

$$\begin{aligned} & -\left(\left(\operatorname{Cos}[(c+d x)^2] \operatorname{Cot}[(c+d x)] \left(a+2 a \operatorname{Tan}[(c+d x)^2]+(a+b) \operatorname{Tan}[(c+d x)^4]\right)\right)\right. \\ & \quad \left.\left.(a d \sqrt{a+b \operatorname{Sin}[(c+d x)^4]}\right)\right)+ \\ & \quad \left(\sqrt{a+b} \operatorname{Cos}[(c+d x)] \operatorname{Sin}[(c+d x)] \left(a+2 a \operatorname{Tan}[(c+d x)^2]+(a+b) \operatorname{Tan}[(c+d x)^4]\right)\right)\right. \\ & \quad \left.\left.(a d \sqrt{a+b \operatorname{Sin}[(c+d x)^4]} \left(\sqrt{a}+\sqrt{a+b} \operatorname{Tan}[(c+d x)^2]\right)\right)-\right. \\ & \quad \left.\left.\left((a+b)^{1/4} \operatorname{Cos}[(c+d x)^2] \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{(a+b)^{1/4} \operatorname{Tan}[(c+d x)]}{a^{1/4}}\right], \frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right]\right.\right.\right. \\ & \quad \left.\left.\left.\left(\sqrt{a}+\sqrt{a+b} \operatorname{Tan}[(c+d x)^2]\right) \sqrt{\frac{a+2 a \operatorname{Tan}[(c+d x)^2]+(a+b) \operatorname{Tan}[(c+d x)^4]}{\left(\sqrt{a}+\sqrt{a+b} \operatorname{Tan}[(c+d x)^2]\right)^2}}\right)\right.\right. \\ & \quad \left.\left.\left.\left.(a^{3/4} d \sqrt{a+b \operatorname{Sin}[(c+d x)^4]}\right)+\left(\left(a+b+\sqrt{a} \sqrt{a+b}\right) \operatorname{Cos}[(c+d x)^2]\right.\right.\right.\right. \\ & \quad \left.\left.\left.\left.\operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{(a+b)^{1/4} \operatorname{Tan}[(c+d x)]}{a^{1/4}}\right], \frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right]\right.\right.\right.\right. \\ & \quad \left.\left.\left.\left.\left(\sqrt{a}+\sqrt{a+b} \operatorname{Tan}[(c+d x)^2]\right)\right.\right.\right.\right. \\ & \quad \left.\left.\left.\left.\left.\sqrt{\frac{a+2 a \operatorname{Tan}[(c+d x)^2]+(a+b) \operatorname{Tan}[(c+d x)^4]}{\left(\sqrt{a}+\sqrt{a+b} \operatorname{Tan}[(c+d x)^2]\right)^2}}\right)\right.\right.\right.\right. \end{aligned}$$

Result (type 4, 1403 leaves):

$$\begin{aligned} & -\frac{\sqrt{8 a+3 b-4 b \operatorname{Cos}[2 (c+d x)]+b \operatorname{Cos}[4 (c+d x)]} \operatorname{Cot}[(c+d x)]}{2 \sqrt{2} a d}+ \\ & \quad \left(\pm b \operatorname{Cos}[(c+d x)^2] \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\sqrt{1-\frac{\pm \sqrt{b}}{\sqrt{a}}}\right] \operatorname{Tan}[(c+d x)], \frac{\sqrt{a}+\pm \sqrt{b}}{\sqrt{a}-\pm \sqrt{b}}\right]\right. \\ & \quad \left.\sqrt{1+\left(1-\frac{\pm \sqrt{b}}{\sqrt{a}}\right) \operatorname{Tan}[(c+d x)^2]} \sqrt{1+\left(1+\frac{\pm \sqrt{b}}{\sqrt{a}}\right) \operatorname{Tan}[(c+d x)^2]}\right)\right. \\ & \quad \left.\left(\sqrt{2} a \sqrt{1-\frac{\pm \sqrt{b}}{\sqrt{a}}} d \sqrt{8 a+3 b-4 b \operatorname{Cos}[2 (c+d x)]+b \operatorname{Cos}[4 (c+d x)]}\right)\right)+ \end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{2} \sqrt{2} b \cos[c + dx]^2 \left(\text{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + dx]\right]}{\sqrt{a}}, \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}}\right] - \right. \right. \\
& \quad \left. \left. 2 \text{EllipticPi}\left[\frac{\sqrt{a}}{\sqrt{a} - i \sqrt{b}}, \frac{i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + dx]\right]}{\sqrt{a}}, \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}}\right]\right) \right. \\
& \quad \left. \sqrt{1 + \left(1 - \frac{i \sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} \sqrt{1 + \left(1 + \frac{i \sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} \right) / \\
& \left. \left(a \sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} d \sqrt{8 a + 3 b - 4 b \cos[2(c + dx)] + b \cos[4(c + dx)]} \right) - \right. \\
& \frac{1}{4 a \sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} d (1 + \tan[c + dx]^2)^2 \sqrt{\frac{b \tan[c + dx]^4 + a (1 + \tan[c + dx]^2)^2}{(1 + \tan[c + dx]^2)^2}}} \\
& \left. \left(4 a \sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + dx] + 8 a \sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + dx]^3 + \right. \right. \\
& \quad \left. \left. 4 a \sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + dx]^5 + 4 \sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} b \tan[c + dx]^5 - \right. \right. \\
& \quad \left. \left. 4 i b \text{EllipticPi}\left[\frac{\sqrt{a}}{\sqrt{a} - i \sqrt{b}}, \frac{i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + dx]\right]}{\sqrt{a} - i \sqrt{b}}, \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}}\right]\right. \right. \\
& \quad \left. \left. \sqrt{1 + \left(1 - \frac{i \sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} \sqrt{1 + \left(1 + \frac{i \sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} - \right. \right. \\
& \quad \left. \left. 4 i b \text{EllipticPi}\left[\frac{\sqrt{a}}{\sqrt{a} - i \sqrt{b}}, \frac{i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + dx]\right]}{\sqrt{a} - i \sqrt{b}}, \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}}\right]\right. \right. \\
& \quad \left. \left. \tan[c + dx]^2 \sqrt{1 + \left(1 - \frac{i \sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} \sqrt{1 + \left(1 + \frac{i \sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} + \right. \right. \\
& \quad \left. \left. 4 \sqrt{a} \left(\frac{i \sqrt{a} + \sqrt{b}}{\sqrt{a}}\right) \text{EllipticE}\left[\frac{i \operatorname{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c + dx]\right]}{\sqrt{a}}, \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}}\right]\right. \right. \\
& \quad \left. \left. (1 + \tan[c + dx]^2) \sqrt{1 + \left(1 - \frac{i \sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} \sqrt{1 + \left(1 + \frac{i \sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} - \right. \right.
\end{aligned}$$

$$\left(4\sqrt{a} - 3i\sqrt{b}\right)\sqrt{b} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}}} \tan[c + dx]\right], \frac{\sqrt{a} + i\sqrt{b}}{\sqrt{a} - i\sqrt{b}}\right] \\ (1 + \tan[c + dx]^2) \sqrt{1 + \left(1 - \frac{i\sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} \sqrt{1 + \left(1 + \frac{i\sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2}$$

Problem 249: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \sin[x]^5} dx$$

Optimal (type 3, 384 leaves, 17 steps):

$$\frac{2 \text{ArcTan}\left[\frac{b^{1/5} + a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5} - b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} - b^{2/5}}} + \frac{2 \text{ArcTan}\left[\frac{(-1)^{2/5} b^{1/5} + a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5} - (-1)^{4/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} - (-1)^{4/5} b^{2/5}}} + \frac{2 \text{ArcTan}\left[\frac{(-1)^{4/5} b^{1/5} + a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5} + (-1)^{3/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} + (-1)^{3/5} b^{2/5}}} - \\ \frac{2 \text{ArcTan}\left[\frac{(-1)^{3/5} (b^{1/5} + (-1)^{2/5} a^{1/5} \tan\left[\frac{x}{2}\right])}{\sqrt{a^{2/5} + (-1)^{1/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} + (-1)^{1/5} b^{2/5}}} - \frac{2 \text{ArcTan}\left[\frac{(-1)^{1/5} (b^{1/5} + (-1)^{4/5} a^{1/5} \tan\left[\frac{x}{2}\right])}{\sqrt{a^{2/5} - (-1)^{2/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5} - (-1)^{2/5} b^{2/5}}}$$

Result (type 7, 149 leaves):

$$\frac{8}{5} i \text{RootSum}\left[i b - 5 i b \#1^2 + 10 i b \#1^4 + 32 a \#1^5 - 10 i b \#1^6 + 5 i b \#1^8 - i b \#1^{10} \&, \\ 2 \text{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^3 - i \text{Log}\left[1 - 2 \cos[x] \#1 + \#1^2\right] \#1^3 \\ b - 4 b \#1^2 + 16 i a \#1^3 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8\right]$$

Problem 250: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \sin[x]^6} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3} + b^{1/3}} \tan[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3} + b^{1/3}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3} - (-1)^{1/3} b^{1/3}} \tan[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3} - (-1)^{1/3} b^{1/3}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3} + (-1)^{2/3} b^{1/3}} \tan[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3} + (-1)^{2/3} b^{1/3}}}$$

Result (type 7, 148 leaves):

$$-\frac{8}{3} \text{RootSum}\left[b - 6 b \#1 + 15 b \#1^2 - 64 a \#1^3 - 20 b \#1^3 + 15 b \#1^4 - 6 b \#1^5 + b \#1^6 \&, \\ 2 \text{ArcTan}\left[\frac{\sin[2x]}{\cos[2x] - \#1}\right] \#1^2 - i \text{Log}\left[1 - 2 \cos[2x] \#1 + \#1^2\right] \#1^2 \\ - b + 5 b \#1 - 32 a \#1^2 - 10 b \#1^2 + 10 b \#1^3 - 5 b \#1^4 + b \#1^5\right]$$

Problem 251: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \sin[x]^8} dx$$

Optimal (type 3, 245 leaves, 9 steps) :

$$\begin{aligned} & -\frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}-b^{1/4}} \tan[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}-b^{1/4}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}-i b^{1/4}} \tan[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}-i b^{1/4}}} - \\ & \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}+i b^{1/4}} \tan[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}+i b^{1/4}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}+b^{1/4}} \tan[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}+b^{1/4}}} \end{aligned}$$

Result (type 7, 174 leaves) :

$$\begin{aligned} & 8 \operatorname{RootSum}\left[b-8 b^{\#1}+28 b^{\#1^2}-56 b^{\#1^3}+256 a^{\#1^4}+70 b^{\#1^4}-56 b^{\#1^5}+28 b^{\#1^6}-8 b^{\#1^7}+b^{\#1^8} \&, \right. \\ & \left. \left(2 \operatorname{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right]^{\#1^3}-i \log\left[1-2 \cos[2x]^{\#1}+\#1^2\right]^{\#1^3}\right) / \right. \\ & \left. \left(-b+7 b^{\#1}-21 b^{\#1^2}+128 a^{\#1^3}+35 b^{\#1^3}-35 b^{\#1^4}+21 b^{\#1^5}-7 b^{\#1^6}+b^{\#1^7}\right) \&\right] \end{aligned}$$

Problem 252: Result is not expressed in closed-form.

$$\int \frac{1}{a - b \sin[x]^5} dx$$

Optimal (type 3, 379 leaves, 17 steps) :

$$\begin{aligned} & -\frac{2 \operatorname{ArcTan}\left[\frac{b^{1/5}-a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}-b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}-b^{2/5}}} - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{2/5} b^{1/5}-a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}-(-1)^{4/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}-(-1)^{4/5} b^{2/5}}} - \\ & \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{4/5} b^{1/5}-a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}+(-1)^{3/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}+(-1)^{3/5} b^{2/5}}} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/5} b^{1/5}+a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}-(-1)^{2/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}-(-1)^{2/5} b^{2/5}}} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{3/5} b^{1/5}+a^{1/5} \tan\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}+(-1)^{1/5} b^{2/5}}}\right]}{5 a^{4/5} \sqrt{a^{2/5}+(-1)^{1/5} b^{2/5}}} \end{aligned}$$

Result (type 7, 149 leaves) :

$$\begin{aligned} & -\frac{8}{5} i \operatorname{RootSum}\left[-i b+5 i b^{\#1^2}-10 i b^{\#1^4}+32 a^{\#1^5}+10 i b^{\#1^6}-5 i b^{\#1^8}+i b^{\#1^{10}} \&, \right. \\ & \left. 2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right]^{\#1^3}-i \log\left[1-2 \cos[x]^{\#1}+\#1^2\right]^{\#1^3}\right) / \& \\ & b-4 b^{\#1^2}-16 i a^{\#1^3}+6 b^{\#1^4}-4 b^{\#1^6}+b^{\#1^8} \end{aligned}$$

Problem 253: Result is not expressed in closed-form.

$$\int \frac{1}{a - b \sin[x]^6} dx$$

Optimal (type 3, 175 leaves, 7 steps) :

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}-b^{1/3}} \tan[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-b^{1/3}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+(-1)^{1/3} b^{1/3}} \tan[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}+(-1)^{1/3} b^{1/3}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}-(-1)^{2/3} b^{1/3}} \tan[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-(-1)^{2/3} b^{1/3}}}$$

Result (type 7, 148 leaves) :

$$\begin{aligned} & \frac{8}{3} \text{RootSum}\left[b - 6 b^{\#1} + 15 b^{\#1^2} + 64 a^{\#1^3} - 20 b^{\#1^3} + 15 b^{\#1^4} - 6 b^{\#1^5} + b^{\#1^6} \&, \right. \\ & \quad \left. 2 \text{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right]^{\#1^2} - i \log\left[1 - 2 \cos[2x]^{\#1} + \#1^2\right]^{\#1^2} \& \right] \\ & \quad - b + 5 b^{\#1} + 32 a^{\#1^2} - 10 b^{\#1^2} + 10 b^{\#1^3} - 5 b^{\#1^4} + b^{\#1^5} \end{aligned}$$

Problem 254: Result is not expressed in closed-form.

$$\int \frac{1}{a - b \sin[x]^8} dx$$

Optimal (type 3, 213 leaves, 9 steps) :

$$\begin{aligned} & \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}-b^{1/4}} \tan[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}-b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}-i b^{1/4}} \tan[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}-i b^{1/4}}} + \\ & \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}+i b^{1/4}} \tan[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}+i b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/4}+b^{1/4}} \tan[x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}+b^{1/4}}} \end{aligned}$$

Result (type 7, 174 leaves) :

$$\begin{aligned} & -8 \text{RootSum}\left[b - 8 b^{\#1} + 28 b^{\#1^2} - 56 b^{\#1^3} - 256 a^{\#1^4} + 70 b^{\#1^4} - 56 b^{\#1^5} + 28 b^{\#1^6} - 8 b^{\#1^7} + b^{\#1^8} \&, \right. \\ & \quad \left. \left(2 \text{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right]^{\#1^3} - i \log\left[1 - 2 \cos[2x]^{\#1} + \#1^2\right]^{\#1^3}\right) / \right. \\ & \quad \left. (-b + 7 b^{\#1} - 21 b^{\#1^2} - 128 a^{\#1^3} + 35 b^{\#1^3} - 35 b^{\#1^4} + 21 b^{\#1^5} - 7 b^{\#1^6} + b^{\#1^7}) \& \right] \end{aligned}$$

Problem 255: Result is not expressed in closed-form.

$$\int \frac{1}{1 + \sin[x]^5} dx$$

Optimal (type 3, 195 leaves, 15 steps) :

$$\begin{aligned} & \frac{2 \text{ArcTan}\left[\frac{(-1)^{2/5}+\tan\left[\frac{x}{2}\right]}{\sqrt{1-(-1)^{4/5}}}\right]}{5 \sqrt{1-\left(-1\right)^{4/5}}} + \frac{2 \text{ArcTan}\left[\frac{(-1)^{4/5}+\tan\left[\frac{x}{2}\right]}{\sqrt{1+(-1)^{3/5}}}\right]}{5 \sqrt{1+\left(-1\right)^{3/5}}} - \\ & \frac{2 \text{ArcTan}\left[\frac{(-1)^{3/5} \left(1+(-1)^{2/5} \tan\left[\frac{x}{2}\right]\right)}{\sqrt{1+(-1)^{1/5}}}\right]}{5 \sqrt{1+\left(-1\right)^{1/5}}} - \frac{2 \text{ArcTan}\left[\frac{(-1)^{1/5} \left(1+(-1)^{4/5} \tan\left[\frac{x}{2}\right]\right)}{\sqrt{1-(-1)^{2/5}}}\right]}{5 \sqrt{1-\left(-1\right)^{2/5}}} - \frac{\cos[x]}{5 \left(1+\sin[x]\right)} \end{aligned}$$

Result (type 7, 411 leaves) :

$$\begin{aligned}
 & -\frac{1}{10} i \operatorname{RootSum}\left[1+2 i \#1-8 \#1^2-14 i \#1^3+30 \#1^4+14 i \#1^5-8 \#1^6-2 i \#1^7+\#1^8 \&, \right. \\
 & \quad \frac{1}{i-8 \#1-21 i \#1^2+60 \#1^3+35 i \#1^4-24 \#1^5-7 i \#1^6+4 \#1^7} \\
 & \quad \left(-2 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\#1}\right]+i \log \left[1-2 \cos [x] \#1+\#1^2\right]-8 i \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\#1}\right] \#1-\right. \\
 & \quad \left.4 \log \left[1-2 \cos [x] \#1+\#1^2\right] \#1+30 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\#1}\right] \#1^2-\right. \\
 & \quad \left.15 i \log \left[1-2 \cos [x] \#1+\#1^2\right] \#1^2+80 i \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\#1}\right] \#1^3+\right. \\
 & \quad \left.40 \log \left[1-2 \cos [x] \#1+\#1^2\right] \#1^3-30 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\#1}\right] \#1^4+\right. \\
 & \quad \left.15 i \log \left[1-2 \cos [x] \#1+\#1^2\right] \#1^4-8 i \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\#1}\right] \#1^5-\right. \\
 & \quad \left.4 \log \left[1-2 \cos [x] \#1+\#1^2\right] \#1^5+2 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\#1}\right] \#1^6-\right. \\
 & \quad \left.i \log \left[1-2 \cos [x] \#1+\#1^2\right] \#1^6\right)\&]+\frac{2 \sin \left[\frac{x}{2}\right]}{5 \left(\cos \left[\frac{x}{2}\right]+\sin \left[\frac{x}{2}\right]\right)}
 \end{aligned}$$

Problem 257: Result is not expressed in closed-form.

$$\int \frac{1}{1 + \sin[x]^8} dx$$

Optimal (type 3, 218 leaves, 9 steps) :

$$\begin{aligned}
 & \frac{1}{8} \left(\sqrt{1+\sqrt{4-2 \sqrt{2}}}+\sqrt{2+2 \times 2^{1/4}+2 \sqrt{1+\sqrt{2}}}+2 \sqrt{2+\sqrt{2}}+\sqrt{1+\sqrt{4+2 \sqrt{2}}} \right) \\
 & (x-\operatorname{ArcTan}[\tan [x]])+\frac{\operatorname{ArcTan}\left[\sqrt{1-(-1)^{1/4}} \tan [x]\right]}{4 \sqrt{1-(-1)^{1/4}}}+\frac{\operatorname{ArcTan}\left[\sqrt{1+(-1)^{1/4}} \tan [x]\right]}{4 \sqrt{1+(-1)^{1/4}}}+ \\
 & \frac{\operatorname{ArcTan}\left[\sqrt{1-(-1)^{3/4}} \tan [x]\right]}{4 \sqrt{1-(-1)^{3/4}}}+\frac{\operatorname{ArcTan}\left[\sqrt{1+(-1)^{3/4}} \tan [x]\right]}{4 \sqrt{1+(-1)^{3/4}}}
 \end{aligned}$$

Result (type 7, 141 leaves) :

$$\begin{aligned}
 & 8 \operatorname{RootSum}\left[1-8 \#1+28 \#1^2-56 \#1^3+326 \#1^4-56 \#1^5+28 \#1^6-8 \#1^7+\#1^8 \&, \right. \\
 & \quad \frac{2 \operatorname{ArcTan}\left[\frac{\sin [2 x]}{\cos [2 x]-\#1}\right] \#1^3-i \log \left[1-2 \cos [2 x] \#1+\#1^2\right] \#1^3}{-1+7 \#1-21 \#1^2+163 \#1^3-35 \#1^4+21 \#1^5-7 \#1^6+\#1^7}\&]
 \end{aligned}$$

Problem 258: Result is not expressed in closed-form.

$$\int \frac{1}{1 - \sin[x]^5} dx$$

Optimal (type 3, 187 leaves, 15 steps):

$$\begin{aligned} & -\frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{2/5}-\tan\left[\frac{x}{2}\right]}{\sqrt{1-(-1)^{4/5}}}\right]}{5 \sqrt{1-(-1)^{4/5}}} - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{4/5}-\tan\left[\frac{x}{2}\right]}{\sqrt{1+(-1)^{3/5}}}\right]}{5 \sqrt{1+(-1)^{3/5}}} + \\ & \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/5}+\tan\left[\frac{x}{2}\right]}{\sqrt{1-(-1)^{2/5}}}\right]}{5 \sqrt{1-(-1)^{2/5}}} + \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{3/5}+\tan\left[\frac{x}{2}\right]}{\sqrt{1+(-1)^{1/5}}}\right]}{5 \sqrt{1+(-1)^{1/5}}} + \frac{\cos[x]}{5 (1-\sin[x])} \end{aligned}$$

Result (type 7, 413 leaves):

$$\begin{aligned} & \frac{1}{10} \operatorname{iRootSum}\left[1 - 2 \operatorname{i}\#1 - 8 \operatorname{\#1^2} + 14 \operatorname{i}\#1^3 + 30 \operatorname{\#1^4} - 14 \operatorname{i}\#1^5 - 8 \operatorname{\#1^6} + 2 \operatorname{i}\#1^7 + \operatorname{\#1^8} \&, \right. \\ & \frac{1}{- \operatorname{i} - 8 \operatorname{\#1} + 21 \operatorname{i}\#1^2 + 60 \operatorname{\#1^3} - 35 \operatorname{i}\#1^4 - 24 \operatorname{\#1^5} + 7 \operatorname{i}\#1^6 + 4 \operatorname{\#1^7}} \\ & \left(-2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \operatorname{\#1}}\right] + \operatorname{i} \operatorname{Log}\left[1 - 2 \cos[x] \operatorname{\#1} + \operatorname{\#1^2}\right] + 8 \operatorname{i} \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \operatorname{\#1}}\right] \operatorname{\#1} + \right. \\ & 4 \operatorname{Log}\left[1 - 2 \cos[x] \operatorname{\#1} + \operatorname{\#1^2}\right] \operatorname{\#1} + 30 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \operatorname{\#1}}\right] \operatorname{\#1^2} - \\ & 15 \operatorname{i} \operatorname{Log}\left[1 - 2 \cos[x] \operatorname{\#1} + \operatorname{\#1^2}\right] \operatorname{\#1^2} - 80 \operatorname{i} \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \operatorname{\#1}}\right] \operatorname{\#1^3} - \\ & 40 \operatorname{Log}\left[1 - 2 \cos[x] \operatorname{\#1} + \operatorname{\#1^2}\right] \operatorname{\#1^3} - 30 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \operatorname{\#1}}\right] \operatorname{\#1^4} + \\ & 15 \operatorname{i} \operatorname{Log}\left[1 - 2 \cos[x] \operatorname{\#1} + \operatorname{\#1^2}\right] \operatorname{\#1^4} + 8 \operatorname{i} \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \operatorname{\#1}}\right] \operatorname{\#1^5} + \\ & 4 \operatorname{Log}\left[1 - 2 \cos[x] \operatorname{\#1} + \operatorname{\#1^2}\right] \operatorname{\#1^5} + 2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \operatorname{\#1}}\right] \operatorname{\#1^6} - \\ & \left. \operatorname{i} \operatorname{Log}\left[1 - 2 \cos[x] \operatorname{\#1} + \operatorname{\#1^2}\right] \operatorname{\#1^6} \right) \&] + \frac{2 \sin\left[\frac{x}{2}\right]}{5 \left(\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right)} \end{aligned}$$

Problem 265: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[x]}{a - a \sin[x]^2} dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sin[x]]}{a}$$

Result (type 3, 37 leaves):

$$\frac{-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]-\sin\left[\frac{x}{2}\right]\right]+\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]+\sin\left[\frac{x}{2}\right]\right]}{a}$$

Problem 270: Result more than twice size of optimal antiderivative.

$$\int \frac{\sec[x]}{a-a \sin[x]^2} dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}[\sin[x]]}{2 a}+\frac{\sec [x] \tan [x]}{2 a}$$

Result (type 3, 45 leaves):

$$\frac{-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]-\sin\left[\frac{x}{2}\right]\right]+\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]+\sin\left[\frac{x}{2}\right]\right]+\sec [x] \tan [x]}{2 a}$$

Problem 276: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[x]^3}{(a-a \sin[x]^2)^2} dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sin[x]]}{a^2}$$

Result (type 3, 37 leaves):

$$\frac{-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]-\sin\left[\frac{x}{2}\right]\right]+\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]+\sin\left[\frac{x}{2}\right]\right]}{a^2}$$

Problem 277: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[x]}{(a-a \sin[x]^2)^2} dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}[\sin[x]]}{2 a^2}+\frac{\sec [x] \tan [x]}{2 a^2}$$

Result (type 3, 45 leaves):

$$\frac{-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]-\sin\left[\frac{x}{2}\right]\right]+\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]+\sin\left[\frac{x}{2}\right]\right]+\sec [x] \tan [x]}{2 a^2}$$

Problem 291: Result more than twice size of optimal antiderivative.

$$\int \sec [e + f x]^6 (a + b \sin [e + f x]^2) dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$\frac{a \tan [e + f x]}{f} + \frac{(2 a + b) \tan [e + f x]^3}{3 f} + \frac{(a + b) \tan [e + f x]^5}{5 f}$$

Result (type 3, 117 leaves):

$$\begin{aligned} & \frac{8 a \tan [e + f x]}{15 f} - \frac{2 b \tan [e + f x]}{15 f} + \frac{4 a \sec [e + f x]^2 \tan [e + f x]}{15 f} - \\ & \frac{b \sec [e + f x]^2 \tan [e + f x]}{15 f} + \frac{a \sec [e + f x]^4 \tan [e + f x]}{5 f} + \frac{b \sec [e + f x]^4 \tan [e + f x]}{5 f} \end{aligned}$$

Problem 292: Result more than twice size of optimal antiderivative.

$$\int \sec [e + f x]^8 (a + b \sin [e + f x]^2) dx$$

Optimal (type 3, 72 leaves, 3 steps):

$$\frac{a \tan [e + f x]}{f} + \frac{(3 a + b) \tan [e + f x]^3}{3 f} + \frac{(3 a + 2 b) \tan [e + f x]^5}{5 f} + \frac{(a + b) \tan [e + f x]^7}{7 f}$$

Result (type 3, 161 leaves):

$$\begin{aligned} & \frac{16 a \tan [e + f x]}{35 f} - \frac{8 b \tan [e + f x]}{105 f} + \frac{8 a \sec [e + f x]^2 \tan [e + f x]}{35 f} - \\ & \frac{4 b \sec [e + f x]^2 \tan [e + f x]}{105 f} + \frac{6 a \sec [e + f x]^4 \tan [e + f x]}{35 f} - \\ & \frac{b \sec [e + f x]^4 \tan [e + f x]}{35 f} + \frac{a \sec [e + f x]^6 \tan [e + f x]}{7 f} + \frac{b \sec [e + f x]^6 \tan [e + f x]}{7 f} \end{aligned}$$

Problem 294: Result unnecessarily involves imaginary or complex numbers.

$$\int \cos [e + f x]^2 (a + b \sin [e + f x]^2)^2 dx$$

Optimal (type 3, 116 leaves, 5 steps):

$$\begin{aligned} & \frac{1}{16} (8 a^2 + 4 a b + b^2) x + \frac{(8 a^2 + 4 a b + b^2) \cos [e + f x] \sin [e + f x]}{16 f} - \\ & \frac{b (8 a + 3 b) \cos [e + f x]^3 \sin [e + f x]}{24 f} - \frac{b \cos [e + f x]^5 \sin [e + f x] (a + (a + b) \tan [e + f x]^2)}{6 f} \end{aligned}$$

Result (type 3, 79 leaves):

$$\frac{1}{192 f} \left(12 ((2 - 2 i) a + b) ((2 + 2 i) a + b) (e + f x) + 3 (4 a - b) (4 a + b) \sin[2 (e + f x)] - 3 b (4 a + b) \sin[4 (e + f x)] + b^2 \sin[6 (e + f x)] \right)$$

Problem 308: Result more than twice size of optimal antiderivative.

$$\int \frac{\sec[x]}{a + b \sin[x]^2} dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a} (a + b)} + \frac{\operatorname{ArcTanh}[\sin[x]]}{a + b}$$

Result (type 3, 96 leaves):

$$\frac{1}{2 \sqrt{a} (a + b)} \left(-\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \csc[x]}{\sqrt{b}}\right] + \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right] + 2 \sqrt{a} \left(-\operatorname{Log}[\cos[\frac{x}{2}]] - \sin[\frac{x}{2}] \right) + \operatorname{Log}[\cos[\frac{x}{2}]] + \sin[\frac{x}{2}] \right)$$

Problem 310: Result more than twice size of optimal antiderivative.

$$\int \frac{\sec[x]^3}{a + b \sin[x]^2} dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$\frac{b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a} (a + b)^2} + \frac{(a + 3 b) \operatorname{ArcTanh}[\sin[x]]}{2 (a + b)^2} + \frac{\sec[x] \tan[x]}{2 (a + b)}$$

Result (type 3, 147 leaves):

$$\frac{1}{4 (a + b)^2} \left(-\frac{2 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \csc[x]}{\sqrt{b}}\right]}{\sqrt{a}} + \frac{2 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a}} - 2 (a + 3 b) \operatorname{Log}[\cos[\frac{x}{2}]] - \sin[\frac{x}{2}] + 2 (a + 3 b) \operatorname{Log}[\cos[\frac{x}{2}]] + \sin[\frac{x}{2}] + \frac{a + b}{(\cos[\frac{x}{2}] - \sin[\frac{x}{2}])^2} - \frac{a + b}{(\cos[\frac{x}{2}] + \sin[\frac{x}{2}])^2} \right)$$

Problem 312: Result more than twice size of optimal antiderivative.

$$\int \frac{\sec[x]^5}{a + b \sin[x]^2} dx$$

Optimal (type 3, 93 leaves, 6 steps) :

$$\begin{aligned} & \frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a} (a+b)^3} + \frac{(3 a^2 + 10 a b + 15 b^2) \operatorname{ArcTanh}[\sin[x]]}{8 (a+b)^3} + \\ & \frac{(3 a + 7 b) \sec[x] \tan[x]}{8 (a+b)^2} + \frac{\sec[x]^3 \tan[x]}{4 (a+b)} \end{aligned}$$

Result (type 3, 214 leaves) :

$$\begin{aligned} & -\frac{1}{16 (a+b)^3} \left(\frac{8 b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \csc[x]}{\sqrt{b}}\right]}{\sqrt{a}} - \frac{8 b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sin[x]}{\sqrt{a}}\right]}{\sqrt{a}} + \right. \\ & 2 (3 a^2 + 10 a b + 15 b^2) \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - 2 (3 a^2 + 10 a b + 15 b^2) \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - \\ & \left. \frac{(a+b)^2}{\left(\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right)^4} + \frac{(a+b)^2}{\left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right)^4} + \frac{(a+b) (3 a + 7 b)}{\left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right)^2} + \frac{(a+b) (3 a + 7 b)}{-1 + \sin[x]} \right) \end{aligned}$$

Problem 373: Result unnecessarily involves higher level functions.

$$\int \cos[e+f x]^5 (a+b \sin[e+f x]^2)^p dx$$

Optimal (type 5, 214 leaves, 5 steps) :

$$\begin{aligned} & -\frac{(3 a + b (7 + 2 p)) \sin[e+f x] (a + b \sin[e+f x]^2)^{1+p}}{b^2 f (3 + 2 p) (5 + 2 p)} - \\ & \frac{\cos[e+f x]^2 \sin[e+f x] (a + b \sin[e+f x]^2)^{1+p}}{b f (5 + 2 p)} + \\ & \left((3 a^2 + 2 a b (5 + 2 p) + b^2 (15 + 16 p + 4 p^2)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \sin[e+f x]^2}{a}\right] \right. \\ & \left. \sin[e+f x] (a + b \sin[e+f x]^2)^p \left(1 + \frac{b \sin[e+f x]^2}{a}\right)^{-p} \right) / (b^2 f (3 + 2 p) (5 + 2 p)) \end{aligned}$$

Result (type 6, 191 leaves) :

$$\begin{aligned} & \left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, -2, -p, \frac{3}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] \right. \\ & \left. \cos[e+f x]^4 \sin[e+f x] (a + b \sin[e+f x]^2)^p \right) / \\ & \left(f \left(3 a \operatorname{AppellF1}\left[\frac{1}{2}, -2, -p, \frac{3}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] + \right. \right. \\ & \left. \left. 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, -2, 1-p, \frac{5}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] - \right. \right. \\ & \left. \left. 2 a \operatorname{AppellF1}\left[\frac{3}{2}, -1, -p, \frac{5}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] \right) \sin[e+f x]^2 \right) \right) \end{aligned}$$

Problem 374: Unable to integrate problem.

$$\int \cos[e + fx]^3 (a + b \sin[e + fx]^2)^p dx$$

Optimal (type 5, 124 leaves, 4 steps):

$$\begin{aligned} & -\frac{\sin[e + fx] (a + b \sin[e + fx]^2)^{1+p}}{b f (3 + 2 p)} + \frac{1}{b f (3 + 2 p)} \\ & (a + b (3 + 2 p)) \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \sin[e + fx]^2}{a}\right] \\ & \sin[e + fx] (a + b \sin[e + fx]^2)^p \left(1 + \frac{b \sin[e + fx]^2}{a}\right)^{-p} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \sec[e + fx]^3 (a + b \sin[e + fx]^2)^p dx$$

Problem 376: Unable to integrate problem.

$$\int \sec[e + fx] (a + b \sin[e + fx]^2)^p dx$$

Optimal (type 6, 76 leaves, 3 steps):

$$\begin{aligned} & \frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] \\ & \sin[e + fx] (a + b \sin[e + fx]^2)^p \left(1 + \frac{b \sin[e + fx]^2}{a}\right)^{-p} \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \sec[e + fx] (a + b \sin[e + fx]^2)^p dx$$

Problem 377: Unable to integrate problem.

$$\int \sec[e + fx]^3 (a + b \sin[e + fx]^2)^p dx$$

Optimal (type 6, 76 leaves, 3 steps):

$$\begin{aligned} & \frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] \\ & \sin[e + fx] (a + b \sin[e + fx]^2)^p \left(1 + \frac{b \sin[e + fx]^2}{a}\right)^{-p} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \sec[e + fx]^3 (a + b \sin[e + fx]^2)^p dx$$

Problem 378: Result more than twice size of optimal antiderivative.

$$\int \cos[e + fx]^4 (a + b \sin[e + fx]^2)^p dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -p, \frac{3}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] \\ \sqrt{\cos[e + fx]^2} (a + b \sin[e + fx]^2)^p \left(1 + \frac{b \sin[e + fx]^2}{a}\right)^{-p} \tan[e + fx]$$

Result (type 6, 199 leaves):

$$\left(3 a \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -p, \frac{3}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] \right. \\ \left. \cos[e + fx]^3 \sin[e + fx] (a + b \sin[e + fx]^2)^p\right) / \\ \left(f \left(3 a \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -p, \frac{3}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] + \right. \right. \\ \left. \left(2 b p \text{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}, 1-p, \frac{5}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] - \right. \right. \\ \left. \left. 3 a \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -p, \frac{5}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right]\right) \sin[e + fx]^2\right)$$

Problem 379: Result more than twice size of optimal antiderivative.

$$\int \cos[e + fx]^2 (a + b \sin[e + fx]^2)^p dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] \\ \sqrt{\cos[e + fx]^2} (a + b \sin[e + fx]^2)^p \left(1 + \frac{b \sin[e + fx]^2}{a}\right)^{-p} \tan[e + fx]$$

Result (type 6, 195 leaves):

$$\left(3 a \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] (a + b \sin[e + fx]^2)^p \right. \\ \left. \sin[2(e + fx)]\right) / \left(2 f \left(3 a \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] + \right. \right. \\ \left. \left(2 b p \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right] - \right. \right. \\ \left. \left. a \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, \sin[e + fx]^2, -\frac{b \sin[e + fx]^2}{a}\right]\right) \sin[e + fx]^2\right)$$

Problem 381: Unable to integrate problem.

$$\int \sec(e + fx)^2 (a + b \sin(e + fx)^2)^p dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \\ \sqrt{\cos[e+fx]^2} (a + b \sin[e+fx]^2)^p \left(1 + \frac{b \sin[e+fx]^2}{a}\right)^{-p} \tan[e+fx]$$

Result (type 8, 25 leaves):

$$\int \sec(e + fx)^2 (a + b \sin(e + fx)^2)^p dx$$

Problem 382: Unable to integrate problem.

$$\int \sec(e + fx)^4 (a + b \sin(e + fx)^2)^p dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, \sin[e+fx]^2, -\frac{b \sin[e+fx]^2}{a}\right] \\ \sqrt{\cos[e+fx]^2} (a + b \sin[e+fx]^2)^p \left(1 + \frac{b \sin[e+fx]^2}{a}\right)^{-p} \tan[e+fx]$$

Result (type 8, 25 leaves):

$$\int \sec(e + fx)^4 (a + b \sin(e + fx)^2)^p dx$$

Problem 383: Result is not expressed in closed-form.

$$\int \frac{\cos[c+dx]^5}{a+b \sin[c+dx]^3} dx$$

Optimal (type 3, 219 leaves, 11 steps):

$$\frac{(a^{4/3} - b^{4/3}) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} \sin[c+dx]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{5/3} d} + \frac{(a^{4/3} + b^{4/3}) \log[a^{1/3} + b^{1/3} \sin[c+dx]]}{3 a^{2/3} b^{5/3} d} - \\ \frac{(a^{4/3} + b^{4/3}) \log[a^{2/3} - a^{1/3} b^{1/3} \sin[c+dx] + b^{2/3} \sin[c+dx]^2]}{6 a^{2/3} b^{5/3} d} - \\ \frac{2 \log[a + b \sin[c+dx]^3]}{3 b d} + \frac{\sin[c+dx]^2}{2 b d}$$

Result (type 7, 230 leaves):

$$\frac{1}{12 b d} \left(-3 \cos[2(c + d x)] + 24 \log[\sec[\frac{1}{2}(c + d x)]^2] - 4 \text{RootSum}[a + 3 a \#1^2 + 8 b \#1^3 + 3 a \#1^4 + a \#1^6 \&, \right.$$

$$\left. \left(-b \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] + 4 a \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1 + \right. \right.$$

$$\left. \left. 8 b \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1^2 + 2 a \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1^3 + \right. \right.$$

$$\left. \left. b \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1^4 + 2 a \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1^5 \right) \right) / \\ (a \#1 + 4 b \#1^2 + 2 a \#1^3 + a \#1^5) \&]$$

Problem 384: Result is not expressed in closed-form.

$$\int \frac{\cos[c + d x]^3}{a + b \sin[c + d x]^3} dx$$

Optimal (type 3, 167 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \sin[c+d x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{1/3} d} + \frac{\log\left[a^{1/3}+b^{1/3} \sin[c+d x]\right]}{3 a^{2/3} b^{1/3} d} -$$

$$\frac{\log\left[a^{2/3}-a^{1/3} b^{1/3} \sin[c+d x]+b^{2/3} \sin[c+d x]^2\right]}{6 a^{2/3} b^{1/3} d} - \frac{\log\left[a+b \sin[c+d x]^3\right]}{3 b d}$$

Result (type 7, 216 leaves):

$$-\frac{1}{3 b d} \left(-3 \log[\sec[\frac{1}{2}(c + d x)]^2] + \right.$$

$$\left. \text{RootSum}[a + 3 a \#1^2 + 8 b \#1^3 + 3 a \#1^4 + a \#1^6 \&, \left(-b \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] + \right. \right.$$

$$\left. \left. a \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1 + 4 b \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1^2 + \right. \right.$$

$$\left. \left. 2 a \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1^3 + b \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1^4 + \right. \right.$$

$$\left. \left. a \log[-\#1 + \tan[\frac{1}{2}(c + d x)]] \#1^5 \right) \right) / (a \#1 + 4 b \#1^2 + 2 a \#1^3 + a \#1^5) \&]$$

Problem 386: Result is not expressed in closed-form.

$$\int \frac{\sec[c + d x]}{a + b \sin[c + d x]^3} dx$$

Optimal (type 3, 290 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b^{1/3} (a^{4/3} - b^{4/3}) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} \sin[c + d x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} (a^2 - b^2) d} - \frac{\operatorname{Log}[1 - \sin[c + d x]]}{2 (a + b) d} + \\
& \frac{\operatorname{Log}[1 + \sin[c + d x]]}{2 (a - b) d} - \frac{b^{1/3} (a^{4/3} + b^{4/3}) \operatorname{Log}\left[a^{1/3} + b^{1/3} \sin[c + d x]\right]}{3 a^{2/3} (a^2 - b^2) d} + \\
& \frac{b^{1/3} (a^{4/3} + b^{4/3}) \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} \sin[c + d x] + b^{2/3} \sin[c + d x]^2\right]}{6 a^{2/3} (a^2 - b^2) d} - \frac{b \operatorname{Log}[a + b \sin[c + d x]^3]}{3 (a^2 - b^2) d}
\end{aligned}$$

Result (type 7, 288 leaves) :

$$\begin{aligned}
& \frac{1}{3 (a - b) (a + b) d} \\
& \left(3 \left(b \operatorname{Log}[\sec\left(\frac{1}{2} (c + d x)\right)^2] + (-a + b) \operatorname{Log}[\cos\left(\frac{1}{2} (c + d x)\right)] - \sin\left(\frac{1}{2} (c + d x)\right) \right) + (a + b) \right. \\
& \quad \left. \operatorname{Log}[\cos\left(\frac{1}{2} (c + d x)\right)] + \sin\left(\frac{1}{2} (c + d x)\right) \right) - \\
& b \operatorname{RootSum}\left[a + 3 a \#1^2 + 8 b \#1^3 + 3 a \#1^4 + a \#1^6 \&, \left(b \operatorname{Log}\left[-\#1 + \tan\left(\frac{1}{2} (c + d x)\right)\right] - \right. \right. \\
& a \operatorname{Log}\left[-\#1 + \tan\left(\frac{1}{2} (c + d x)\right)\right] \#1 + 4 b \operatorname{Log}\left[-\#1 + \tan\left(\frac{1}{2} (c + d x)\right)\right] \#1^2 + \\
& 4 a \operatorname{Log}\left[-\#1 + \tan\left(\frac{1}{2} (c + d x)\right)\right] \#1^3 - b \operatorname{Log}\left[-\#1 + \tan\left(\frac{1}{2} (c + d x)\right)\right] \#1^4 + \\
& \left. \left. a \operatorname{Log}\left[-\#1 + \tan\left(\frac{1}{2} (c + d x)\right)\right] \#1^5 \right) / (a \#1 + 4 b \#1^2 + 2 a \#1^3 + a \#1^5) \& \right]
\end{aligned}$$

Problem 387: Result is not expressed in closed-form.

$$\int \frac{\sec[c + d x]^3}{a + b \sin[c + d x]^3} dx$$

Optimal (type 3, 385 leaves, 11 steps) :

$$\begin{aligned}
& - \frac{b^{5/3} (2 a^2 - 3 a^{4/3} b^{2/3} + b^2) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} \sin[c + d x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} (a^2 - b^2)^2 d} - \frac{(a + 4 b) \operatorname{Log}[1 - \sin[c + d x]]}{4 (a + b)^2 d} + \\
& \frac{(a - 4 b) \operatorname{Log}[1 + \sin[c + d x]]}{4 (a - b)^2 d} + \frac{b^{5/3} (2 a^2 + 3 a^{4/3} b^{2/3} + b^2) \operatorname{Log}\left[a^{1/3} + b^{1/3} \sin[c + d x]\right]}{3 a^{2/3} (a^2 - b^2)^2 d} - \\
& \frac{1}{6 a^{2/3} (a^2 - b^2)^2 d} b^{5/3} (2 a^2 + 3 a^{4/3} b^{2/3} + b^2) \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} \sin[c + d x] + b^{2/3} \sin[c + d x]^2\right] + \\
& \frac{b (a^2 + 2 b^2) \operatorname{Log}[a + b \sin[c + d x]^3]}{3 (a^2 - b^2)^2 d} + \frac{1}{4 (a + b) d (1 - \sin[c + d x])} - \frac{1}{4 (a - b) d (1 + \sin[c + d x])}
\end{aligned}$$

Result (type 7, 535 leaves) :

$$\begin{aligned}
& \frac{1}{12 d} \left(- \frac{6 (a + 4 b) \operatorname{Log}[\cos[\frac{1}{2} (c + d x)] - \sin[\frac{1}{2} (c + d x)]]}{(a + b)^2} + \right. \\
& \frac{6 (a - 4 b) \operatorname{Log}[\cos[\frac{1}{2} (c + d x)] + \sin[\frac{1}{2} (c + d x)]]}{(a - b)^2} + \\
& \frac{1}{(a^2 - b^2)^2} 4 b \left(-3 (a^2 + 2 b^2) \operatorname{Log}[\sec[\frac{1}{2} (c + d x)]^2] + \right. \\
& \operatorname{RootSum}[a + 3 a \#1^2 + 8 b \#1^3 + 3 a \#1^4 + a \#1^6 \&, \frac{1}{a \#1 + 4 b \#1^2 + 2 a \#1^3 + a \#1^5} \\
& \left(2 a^2 b \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] + b^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] + \right. \\
& a^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1 - 4 a b^2 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1 + \\
& 4 a^2 b \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^2 + 8 b^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^2 + \\
& 2 a^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^3 + 10 a b^2 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^3 - \\
& 2 a^2 b \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^4 - b^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^4 + \\
& a^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^5 + 2 a b^2 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^5 \Big) \& \Big) + \\
& \left. \frac{3}{(a + b) (\cos[\frac{1}{2} (c + d x)] - \sin[\frac{1}{2} (c + d x)])^2} - \right. \\
& \left. \frac{3}{(a - b) (\cos[\frac{1}{2} (c + d x)] + \sin[\frac{1}{2} (c + d x)])^2} \right)
\end{aligned}$$

Problem 388: Result is not expressed in closed-form.

$$\int \frac{\cos[c + d x]^4}{a + b \sin[c + d x]^3} dx$$

Optimal (type 3, 764 leaves, 38 steps):

$$\begin{aligned}
& - \frac{2 (-1)^{2/3} a^{2/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/3} b^{1/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} b^{4/3} d} + \\
& \frac{2 \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} d} + \frac{2 a^{2/3} \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^{4/3} d} - \\
& \frac{4 \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^{2/3} d} + \frac{2 \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} - \\
& \frac{2 (-1)^{1/3} a^{2/3} \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} b^{4/3} d} - \frac{2 \operatorname{ArcTan} \left[\frac{(-1)^{1/3} (b^{1/3} + (-1)^{2/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right])}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d} + \\
& \frac{4 \operatorname{ArcTanh} \left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} b^{2/3} d} + \frac{4 \operatorname{ArcTanh} \left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} b^{2/3} d} - \frac{\cos [c + d x]}{b d}
\end{aligned}$$

Result (type 7, 300 leaves):

$$\begin{aligned}
& - \frac{1}{3 b d} \left(3 \cos [c + d x] + \right. \\
& \pm \operatorname{RootSum} \left[-\pm b + 3 \pm b \pm 1^2 + 8 a \pm 1^3 - 3 \pm b \pm 1^4 + \pm b \pm 1^6 \&, \left(2 b \operatorname{ArcTan} \left[\frac{\sin [c + d x]}{\cos [c + d x] - \pm 1} \right] - \right. \right. \\
& \pm b \log \left[1 - 2 \cos [c + d x] \pm 1 + \pm 1^2 \right] - 2 \pm a \operatorname{ArcTan} \left[\frac{\sin [c + d x]}{\cos [c + d x] - \pm 1} \right] \pm 1 - \\
& a \log \left[1 - 2 \cos [c + d x] \pm 1 + \pm 1^2 \right] \pm 1 + 2 \pm a \operatorname{ArcTan} \left[\frac{\sin [c + d x]}{\cos [c + d x] - \pm 1} \right] \pm 1^3 + \\
& a \log \left[1 - 2 \cos [c + d x] \pm 1 + \pm 1^2 \right] \pm 1^3 + 2 b \operatorname{ArcTan} \left[\frac{\sin [c + d x]}{\cos [c + d x] - \pm 1} \right] \pm 1^4 - \\
& \pm b \log \left[1 - 2 \cos [c + d x] \pm 1 + \pm 1^2 \right] \pm 1^4 \Big) \Big/ \left(b \pm 1 - 4 \pm a \pm 1^2 - 2 b \pm 1^3 + b \pm 1^5 \right) \& \Big]
\end{aligned}$$

Problem 389: Result is not expressed in closed-form.

$$\int \frac{\cos [c + d x]^2}{a + b \sin [c + d x]^3} dx$$

Optimal (type 3, 484 leaves, 24 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} d} - \frac{2 \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} b^{2/3} d} + \\
& \frac{2 \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} - \frac{2 \operatorname{ArcTan} \left[\frac{(-1)^{1/3} (b^{1/3} + (-1)^{2/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right])}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d} + \\
& \frac{2 \operatorname{ArcTanh} \left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} b^{2/3} d} + \frac{2 \operatorname{ArcTanh} \left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} b^{2/3} d}
\end{aligned}$$

Result (type 7, 231 leaves) :

$$\begin{aligned}
& -\frac{1}{6 d} \operatorname{RootSum} \left[-b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \right. \\
& \left(2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] - i \operatorname{Log} \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] + 4 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \right. \\
& \left. \#1^2 - 2 i \operatorname{Log} \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^2 + 2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^4 - \right. \\
& \left. i \operatorname{Log} \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^4 \right) / (b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5) \&
\end{aligned}$$

Problem 390: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \sin[c+d x]^3} dx$$

Optimal (type 3, 245 leaves, 11 steps) :

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} d} + \\
& \frac{2 \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} d} - \frac{2 \operatorname{ArcTan} \left[\frac{(-1)^{1/3} (b^{1/3} + (-1)^{2/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right])}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} d}
\end{aligned}$$

Result (type 7, 126 leaves) :

$$\begin{aligned}
& -\frac{1}{3 d} 2 i \operatorname{RootSum} \left[-b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \right. \\
& \left. \frac{2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1 - i \operatorname{Log} \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1}{b - 4 i a \#1 - 2 b \#1^2 + b \#1^4} \&
\right]
\end{aligned}$$

Problem 391: Result is not expressed in closed-form.

$$\int \frac{\sec[c + d x]^2}{a + b \sin[c + d x]^3} dx$$

Optimal (type 3, 299 leaves, ? steps):

$$\begin{aligned} & \frac{2 (-1)^{2/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right] - 2 b^{2/3} \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} (a^{2/3} - (-1)^{2/3} b^{2/3})^{3/2} d} + \\ & \frac{2 (-1)^{1/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right] + \frac{\sec[c + d x] (b - a \sin[c + d x])}{(-a^2 + b^2) d}}{3 a^{2/3} (a^{2/3} + (-1)^{1/3} b^{2/3})^{3/2} d} \end{aligned}$$

Result (type 7, 432 leaves):

$$\begin{aligned} & \left(-6 b + 6 b \cos[c + d x] - i b \cos[c + d x] \right. \\ & \left. \operatorname{RootSum}\left[-i b + 3 i b \#1^2 + 8 a \#1^3 - 3 i b \#1^4 + i b \#1^6 \&, \frac{1}{b \#1 - 4 i a \#1^2 - 2 b \#1^3 + b \#1^5}\right. \right. \\ & \left. \left(2 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] - i b \operatorname{Log}\left[1 - 2 \cos[c + d x] \#1 + \#1^2\right] + \right. \right. \\ & \left. \left. 4 i a \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1 + 2 a \operatorname{Log}\left[1 - 2 \cos[c + d x] \#1 + \#1^2\right] \#1 - \right. \right. \\ & \left. \left. 12 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^2 + 6 i b \operatorname{Log}\left[1 - 2 \cos[c + d x] \#1 + \#1^2\right] \#1^2 - \right. \right. \\ & \left. \left. 4 i a \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^3 - 2 a \operatorname{Log}\left[1 - 2 \cos[c + d x] \#1 + \#1^2\right] \#1^3 + \right. \right. \\ & \left. \left. 2 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 - i b \operatorname{Log}\left[1 - 2 \cos[c + d x] \#1 + \#1^2\right] \#1^4 \right) \& \right] + \\ & 6 a \sin[c + d x] \Bigg) \Bigg/ \left(6 (a - b) (a + b) d \left(\cos\left[\frac{1}{2} (c + d x)\right] - \sin\left[\frac{1}{2} (c + d x)\right] \right) \right. \\ & \left. \left(\cos\left[\frac{1}{2} (c + d x)\right] + \sin\left[\frac{1}{2} (c + d x)\right] \right) \right) \end{aligned}$$

Problem 392: Result is not expressed in closed-form.

$$\int \frac{\sec[c + d x]^4}{a + b \sin[c + d x]^3} dx$$

Optimal (type 3, 1093 leaves, ? steps):

$$\begin{aligned}
& - \frac{2 (-1)^{2/3} a^{2/3} b^{8/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} - \\
& + \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 a^{2/3} b^{8/3} \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{\sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \\
& + \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} - \\
& + \frac{2 (-1)^{1/3} a^{2/3} b^{8/3} \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} + \\
& + \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan} \left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} - \\
& - \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTanh} \left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} - \\
& + \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTanh} \left[\frac{b^{1/3} - (-1)^{2/3} a^{1/3} \tan \left[\frac{1}{2} (c+d x) \right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} + \frac{\cos [c + d x]}{12 (a + b) d (1 - \sin [c + d x])^2} + \\
& - \frac{\cos [c + d x]}{12 (a + b) d (1 - \sin [c + d x])} + \frac{(a + 4 b) \cos [c + d x]}{4 (a + b)^2 d (1 - \sin [c + d x])} - \\
& - \frac{\cos [c + d x]}{12 (a - b) d (1 + \sin [c + d x])^2} - \frac{(a - 4 b) \cos [c + d x]}{4 (a - b)^2 d (1 + \sin [c + d x])} - \frac{\cos [c + d x]}{12 (a - b) d (1 + \sin [c + d x])}
\end{aligned}$$

Result (type 7, 679 leaves):

$$\begin{aligned}
& \frac{1}{24 (a-b)^2 (a+b)^2 d} \\
& \left(4 \pm b^2 \operatorname{RootSum} \left[- \pm b + 3 \pm b \pm 1^2 + 8 a \pm 1^3 - 3 \pm b \pm 1^4 + \pm b \pm 1^6 \&, \frac{1}{b \pm 1 - 4 \pm a \pm 1^2 - 2 b \pm 1^3 + b \pm 1^5} \right. \right. \\
& \left(2 a^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1} \right] + 4 b^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1} \right] - \right. \\
& \pm a^2 \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] - 2 \pm b^2 \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] + \\
& 12 \pm a b \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1} \right] \pm 1 + 6 a b \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1 - \\
& 20 a^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1} \right] \pm 1^2 - 16 b^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1} \right] \pm 1^2 + \\
& 10 \pm a^2 \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^2 + 8 \pm b^2 \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^2 - \\
& 12 \pm a b \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1} \right] \pm 1^3 - 6 a b \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^3 + \\
& 2 a^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1} \right] \pm 1^4 + 4 b^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \pm 1} \right] \pm 1^4 - \\
& \pm a^2 \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^4 - 2 \pm b^2 \log[1 - 2 \cos[c+d x] \pm 1 + \pm 1^2] \pm 1^4 \Big) \& + \\
& \left. \sec[c+d x]^3 \left(4 a^2 b + 32 b^3 - 3 b (5 a^2 + 13 b^2) \cos[c+d x] + 12 b (a^2 + 2 b^2) \cos[2 (c+d x)] - \right. \right. \\
& 5 a^2 b \cos[3 (c+d x)] - 13 b^3 \cos[3 (c+d x)] + 12 a^3 \sin[c+d x] - \\
& \left. \left. 30 a b^2 \sin[c+d x] + 4 a^3 \sin[3 (c+d x)] - 22 a b^2 \sin[3 (c+d x)] \right) \right)
\end{aligned}$$

Problem 393: Result is not expressed in closed-form.

$$\int \frac{\cos[c+d x]^7}{(a+b \sin[c+d x]^3)^2} dx$$

Optimal (type 3, 288 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 (2 a^2 + 3 a^{4/3} b^{2/3} + b^2) \operatorname{ArcTan} \left[\frac{a^{1/3} - 2 b^{1/3} \sin[c+d x]}{\sqrt{3} a^{1/3}} \right]}{3 \sqrt{3} a^{5/3} b^{7/3} d} + \\
& \frac{2 (2 a^2 - 3 a^{4/3} b^{2/3} + b^2) \log[a^{1/3} + b^{1/3} \sin[c+d x]]}{9 a^{5/3} b^{7/3} d} - \frac{1}{9 a^{5/3} b^{7/3} d} \\
& (2 a^2 - 3 a^{4/3} b^{2/3} + b^2) \log[a^{2/3} - a^{1/3} b^{1/3} \sin[c+d x] + b^{2/3} \sin[c+d x]^2] - \\
& \frac{\sin[c+d x]}{b^2 d} - \frac{\sin[c+d x] (a^2 - b^2 + 3 a b \sin[c+d x] + 3 b^2 \sin[c+d x]^2)}{3 a b^2 d (a + b \sin[c+d x]^3)}
\end{aligned}$$

Result (type 7, 490 leaves):

$$\begin{aligned}
& -\frac{1}{9 b^2 d} \left(\frac{1}{a} \operatorname{RootSum} \left[-\frac{1}{2} b + 3 \frac{1}{2} b \#1^2 + 8 a \#1^3 - 3 \frac{1}{2} b \#1^4 + \frac{1}{2} b \#1^6 \&, \frac{1}{b \#1 - 4 \frac{1}{2} a \#1^2 - 2 b \#1^3 + b \#1^5} \right. \right. \\
& \left. \left(-6 a b \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] + 3 \frac{1}{2} a b \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] + \right. \right. \\
& \left. \left. 8 \frac{1}{2} a^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1 + 4 \frac{1}{2} b^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1 + \right. \right. \\
& \left. \left. 4 a^2 \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1 + 2 b^2 \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1 + \right. \right. \\
& \left. \left. 8 \frac{1}{2} a^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^3 + 4 \frac{1}{2} b^2 \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^3 + \right. \right. \\
& \left. \left. 4 a^2 \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^3 + 2 b^2 \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^3 + \right. \right. \\
& \left. \left. 6 a b \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^4 - 3 \frac{1}{2} a b \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^4 \right) \& + \right. \\
& \left. 9 \sin[c+d x] - \frac{6 (3 a b + 3 a b \cos[2 (c+d x)] - 2 (a^2 - b^2) \sin[c+d x])}{a (4 a + 3 b \sin[c+d x] - b \sin[3 (c+d x)])} \right)
\end{aligned}$$

Problem 394: Result is not expressed in closed-form.

$$\int \frac{\cos[c+d x]^5}{(a+b \sin[c+d x]^3)^2} dx$$

Optimal (type 3, 238 leaves, 8 steps):

$$\begin{aligned}
& -\frac{2 (a^{4/3} + b^{4/3}) \operatorname{ArcTan} \left[\frac{a^{1/3} - 2 b^{1/3} \sin[c+d x]}{\sqrt{3} a^{1/3}} \right]}{3 \sqrt{3} a^{5/3} b^{5/3} d} - \frac{2 (a^{4/3} - b^{4/3}) \log[a^{1/3} + b^{1/3} \sin[c+d x]]}{9 a^{5/3} b^{5/3} d} + \\
& \frac{(a^{4/3} - b^{4/3}) \log[a^{2/3} - a^{1/3} b^{1/3} \sin[c+d x] + b^{2/3} \sin[c+d x]^2]}{9 a^{5/3} b^{5/3} d} + \\
& \frac{\sin[c+d x] (b - a \sin[c+d x] - 2 b \sin[c+d x]^2)}{3 a b d (a + b \sin[c+d x]^3)}
\end{aligned}$$

Result (type 7, 346 leaves):

$$\begin{aligned}
& \frac{1}{9 a b d} \left(\frac{1}{2} \operatorname{RootSum} \left[-\frac{1}{2} b + 3 \frac{1}{2} b \#1^2 + 8 a \#1^3 - 3 \frac{1}{2} b \#1^4 + \frac{1}{2} b \#1^6 \&, \right. \right. \\
& \left. \left(-2 \frac{1}{2} a \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] - a \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] - \right. \right. \\
& \left. \left. 4 b \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1 + 2 \frac{1}{2} b \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1 - \right. \right. \\
& \left. \left. 4 b \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^3 + 2 \frac{1}{2} b \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^3 + \right. \right. \\
& \left. \left. 2 \frac{1}{2} a \operatorname{ArcTan} \left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^4 + a \log \left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^4 \right) / \right. \\
& \left. (b \#1 - 4 \frac{1}{2} a \#1^2 - 2 b \#1^3 + b \#1^5) \& + \frac{6 (3 a + a \cos[2 (c+d x)] + 2 b \sin[c+d x])}{4 a + 3 b \sin[c+d x] - b \sin[3 (c+d x)]} \right)
\end{aligned}$$

Problem 395: Result is not expressed in closed-form.

$$\int \frac{\cos[c + dx]^3}{(a + b \sin[c + dx]^3)^2} dx$$

Optimal (type 3, 183 leaves, 9 steps) :

$$\begin{aligned} & -\frac{2 \operatorname{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \sin[c+d x]}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3} b^{1/3} d}+\frac{2 \log \left[a^{1/3}+b^{1/3} \sin [c+d x]\right]}{9 a^{5/3} b^{1/3} d}- \\ & \frac{\log \left[a^{2/3}-a^{1/3} b^{1/3} \sin [c+d x]+b^{2/3} \sin [c+d x]^2\right]}{9 a^{5/3} b^{1/3} d}+\frac{a+b \sin [c+d x]}{3 a b d \left(a+b \sin [c+d x]^3\right)} \end{aligned}$$

Result (type 7, 221 leaves) :

$$\begin{aligned} & \frac{1}{9 a d} 2 \left(-\operatorname{RootSum}\left[-\frac{1}{2} b+3 \frac{1}{2} b \# 1^2+8 a \# 1^3-3 \frac{1}{2} b \# 1^4+\frac{1}{2} b \# 1^6 \&, \right. \right. \\ & \left(2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x]-\# 1}\right]-\frac{1}{2} \log \left[1-2 \cos [c+d x] \# 1+\# 1^2\right]+\right. \\ & \left. \left. 2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x]-\# 1}\right] \# 1^2-\frac{1}{2} \log \left[1-2 \cos [c+d x] \# 1+\# 1^2\right] \# 1^2\right) / \\ & \left. \left(b-4 \frac{1}{2} a \# 1-2 b \# 1^2+b \# 1^4\right) \& \right]+\frac{6 \left(a+b \sin [c+d x]\right)}{b \left(4 a+3 b \sin [c+d x]-b \sin \left[3 \left(c+d x\right)\right]\right)} \right) \end{aligned}$$

Problem 397: Result is not expressed in closed-form.

$$\int \frac{\sec [c + dx]}{(a + b \sin [c + dx]^3)^2} dx$$

Optimal (type 3, 587 leaves, 18 steps) :

$$\begin{aligned} & -\frac{b^{1/3} \left(a^{4/3}-2 b^{4/3}\right) \operatorname{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \sin[c+d x]}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3} \left(a^2-b^2\right) d}-\frac{b^{1/3} \left(a^2-2 a^{2/3} b^{4/3}+b^2\right) \operatorname{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \sin[c+d x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{1/3} \left(a^2-b^2\right)^2 d}- \\ & \frac{\log \left[1-\sin [c+d x]\right]}{2 \left(a+b\right)^2 d}+\frac{\log \left[1+\sin [c+d x]\right]}{2 \left(a-b\right)^2 d}-\frac{b^{1/3} \left(a^{4/3}+2 b^{4/3}\right) \log \left[a^{1/3}+b^{1/3} \sin [c+d x]\right]}{9 a^{5/3} \left(a^2-b^2\right) d}- \\ & \frac{b^{1/3} \left(a^2+2 a^{2/3} b^{4/3}+b^2\right) \log \left[a^{1/3}+b^{1/3} \sin [c+d x]\right]}{3 a^{1/3} \left(a^2-b^2\right)^2 d}+ \\ & \left(b^{1/3} \left(a^{4/3}+2 b^{4/3}\right) \log \left[a^{2/3}-a^{1/3} b^{1/3} \sin [c+d x]+b^{2/3} \sin [c+d x]^2\right]\right) /\left(18 a^{5/3} \left(a^2-b^2\right) d\right)+ \\ & \frac{1}{6 a^{1/3} \left(a^2-b^2\right)^2 d} b^{1/3} \left(a^2+2 a^{2/3} b^{4/3}+b^2\right) \log \left[a^{2/3}-a^{1/3} b^{1/3} \sin [c+d x]+b^{2/3} \sin [c+d x]^2\right]- \\ & \frac{2 a b \log \left[a+b \sin [c+d x]^3\right]}{3 \left(a^2-b^2\right)^2 d}+\frac{b \left(a-\sin [c+d x]\right) \left(b-a \sin [c+d x]\right)}{3 a \left(a^2-b^2\right) d \left(a+b \sin [c+d x]^3\right)} \end{aligned}$$

Result (type 7, 478 leaves) :

$$\begin{aligned}
& \frac{1}{9 d} \left(-\frac{9 \operatorname{Log}[\cos[\frac{1}{2} (c + d x)] - \sin[\frac{1}{2} (c + d x)]]}{(a + b)^2} + \right. \\
& \frac{9 \operatorname{Log}[\cos[\frac{1}{2} (c + d x)] + \sin[\frac{1}{2} (c + d x)]]}{(a - b)^2} + \frac{1}{a (a^2 - b^2)^2} 2 b \left(9 a^2 \operatorname{Log}[\sec[\frac{1}{2} (c + d x)]^2] - \right. \\
& \operatorname{RootSum}[a + 3 a \#1^2 + 8 b \#1^3 + 3 a \#1^4 + a \#1^6 \&, \frac{1}{a \#1 + 4 b \#1^2 + 2 a \#1^3 + a \#1^5} \\
& \left(4 a^2 b \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] - b^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] - \right. \\
& a^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1 - 2 a b^2 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1 + \\
& 12 a^2 b \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^2 + 10 a^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^3 + \\
& 2 a b^2 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^3 - 4 a^2 b \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^4 + \\
& b^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^4 + 3 a^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2} (c + d x)]] \#1^5 \Big) \&] \Big) - \\
& \left. \frac{6 b (-3 a + a \operatorname{Cos}[2 (c + d x)] + 2 b \operatorname{Sin}[c + d x])}{a (a - b) (a + b) (4 a + 3 b \operatorname{Sin}[c + d x] - b \operatorname{Sin}[3 (c + d x)])} \right)
\end{aligned}$$

Problem 398: Result is not expressed in closed-form.

$$\int \frac{\sec[c + d x]^3}{(a + b \operatorname{Sin}[c + d x]^3)^2} dx$$

Optimal (type 3, 747 leaves, 18 steps):

$$\begin{aligned}
& - \frac{b^{5/3} (4 a^2 - 3 a^{4/3} b^{2/3} + 2 b^2) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} \sin[c+d x]}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3} (a^2 - b^2)^2 d} - \\
& \frac{b^{5/3} (4 a^{8/3} - 9 a^2 b^{2/3} + 8 a^{2/3} b^2 - 3 b^{8/3}) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} \sin[c+d x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{1/3} (a^2 - b^2)^3 d} - \frac{(a + 7 b) \log[1 - \sin[c + d x]]}{4 (a + b)^3 d} + \\
& \frac{(a - 7 b) \log[1 + \sin[c + d x]]}{4 (a - b)^3 d} + \frac{b^{5/3} (4 a^2 + 3 a^{4/3} b^{2/3} + 2 b^2) \log[a^{1/3} + b^{1/3} \sin[c + d x]]}{9 a^{5/3} (a^2 - b^2)^2 d} + \\
& \frac{1}{3 a^{1/3} (a^2 - b^2)^3 d} b^{5/3} (3 b^{2/3} (3 a^2 + b^2) + 4 a^{2/3} (a^2 + 2 b^2)) \log[a^{1/3} + b^{1/3} \sin[c + d x]] - \\
& (b^{5/3} (4 a^2 + 3 a^{4/3} b^{2/3} + 2 b^2) \log[a^{2/3} - a^{1/3} b^{1/3} \sin[c + d x] + b^{2/3} \sin[c + d x]^2]) / \\
& (18 a^{5/3} (a^2 - b^2)^2 d) - \frac{1}{6 a^{1/3} (a^2 - b^2)^3 d} \\
& b^{5/3} (3 b^{2/3} (3 a^2 + b^2) + 4 a^{2/3} (a^2 + 2 b^2)) \log[a^{2/3} - a^{1/3} b^{1/3} \sin[c + d x] + b^{2/3} \sin[c + d x]^2] + \\
& \frac{2 a b (a^2 + 5 b^2) \log[a + b \sin[c + d x]^3]}{3 (a^2 - b^2)^3 d} + \frac{1}{4 (a + b)^2 d (1 - \sin[c + d x])} - \\
& \frac{1}{4 (a - b)^2 d (1 + \sin[c + d x])} - \frac{b (a (a^2 + 2 b^2) - b \sin[c + d x] (2 a^2 + b^2 - 3 a b \sin[c + d x]))}{3 a (a^2 - b^2)^2 d (a + b \sin[c + d x]^3)}
\end{aligned}$$

Result (type 7, 773 leaves):

$$\begin{aligned}
& \frac{(-a - 7b) \operatorname{Log}[\cos[\frac{1}{2}(c + dx)] - \sin[\frac{1}{2}(c + dx)]]}{2(a + b)^3 d} + \\
& \frac{(a - 7b) \operatorname{Log}[\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)]]}{2(a - b)^3 d} + \\
& \frac{1}{9a(a^2 - b^2)^3 d} 2b \left(-9a^2(a^2 + 5b^2) \operatorname{Log}[\sec[\frac{1}{2}(c + dx)]^2] + \right. \\
& \operatorname{RootSum}[a + 3a\#1^2 + 8b\#1^3 + 3a\#1^4 + a\#1^6 \&, \frac{1}{a\#1 + 4b\#1^2 + 2a\#1^3 + a\#1^5} \\
& \left(8a^4b \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]] + 11a^2b^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]] - \right. \\
& b^5 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]] + 3a^5 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1 - \\
& 15a^3b^2 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1 - 6ab^4 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1 + \\
& 12a^4b \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^2 + 60a^2b^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^2 + \\
& 6a^5 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^3 + 60a^3b^2 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^3 + \\
& 6ab^4 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^3 - 8a^4b \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^4 - \\
& 11a^2b^3 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^4 + b^5 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^4 + \\
& 3a^5 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^5 + 15a^3b^2 \operatorname{Log}[-\#1 + \tan[\frac{1}{2}(c + dx)]]\#1^5 \Big) \& \Big) + \\
& \frac{1}{4(a + b)^2 d (\cos[\frac{1}{2}(c + dx)] - \sin[\frac{1}{2}(c + dx)])^2} - \\
& \frac{1}{4(a - b)^2 d (\cos[\frac{1}{2}(c + dx)] + \sin[\frac{1}{2}(c + dx)])^2} - \\
& (2 \left((-2a^3b - 7ab^3 + 3ab^3 \cos[2(c + dx)] + 4a^2b^2 \sin[c + dx] + 2b^4 \sin[c + dx]) / \right. \\
& \left. (3a(a - b)^2 (a + b)^2 d (-4a - 3b \sin[c + dx] + b \sin[3(c + dx)])) \right)
\end{aligned}$$

Problem 404: Result is not expressed in closed-form.

$$\int \frac{\cos[c + dx]^7}{a - b \sin[c + dx]^4} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$\frac{\left(\sqrt{a} + \sqrt{b}\right)^3 \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} b^{7/4} d} - \frac{\left(\sqrt{a} - \sqrt{b}\right)^3 \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} b^{7/4} d} - \frac{3 \sin[c+d x]}{b d} + \frac{\sin[c+d x]^3}{3 b d}$$

Result (type 7, 524 leaves) :

$$\frac{1}{24 b d} \left(3 \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \right. \right. \\ \left. \left(-2 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - 6 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + \right. \right. \\ \left. \left. i a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] + 3 i b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] - \right. \right. \\ \left. \left. 22 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 - 2 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + \right. \right. \\ \left. \left. 11 i a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + i b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - \right. \right. \\ \left. \left. 22 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - 2 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 + \right. \right. \\ \left. \left. 11 i a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + i b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 - \right. \right. \\ \left. \left. 2 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - 6 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + \right. \right. \\ \left. \left. i a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 + 3 i b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) \& \right] - \\ \left. 2 \left(33 \sin[c+d x] + \sin[3(c+d x)] \right) \right)$$

Problem 405: Result is not expressed in closed-form.

$$\int \frac{\cos[c+d x]^5}{a - b \sin[c+d x]^4} dx$$

Optimal (type 3, 113 leaves, 6 steps) :

$$\frac{\left(\sqrt{a} + \sqrt{b}\right)^2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} b^{5/4} d} + \frac{\left(a - 2 \sqrt{a} \sqrt{b} + b\right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} b^{5/4} d} - \frac{\sin[c+d x]}{b d}$$

Result (type 7, 411 leaves) :

$$\begin{aligned}
& - \frac{1}{4 b d} \\
& \left(\text{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \right. \right. \\
& \left(2 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] - i b \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] + \right. \\
& \quad 4 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^2 + 2 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^2 - \\
& \quad 2 i a \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^2 - \\
& \quad i b \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^2 + 4 a \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^4 + \\
& \quad 2 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^4 - 2 i a \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^4 - \\
& \quad i b \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^4 + 2 b \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^6 - \\
& \quad i b \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^6 \Big) \& \Big) + 4 \sin[c+d x] \Big)
\end{aligned}$$

Problem 406: Result is not expressed in closed-form.

$$\int \frac{\cos[c+d x]^3}{a - b \sin[c+d x]^4} dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$\frac{\left(\sqrt{a} + \sqrt{b}\right) \text{ArcTan}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} b^{3/4} d} - \frac{\left(\sqrt{a} - \sqrt{b}\right) \text{ArcTanh}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} b^{3/4} d}$$

Result (type 7, 283 leaves):

$$\begin{aligned}
& - \frac{1}{8 d} \text{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \\
& \left(2 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] - i \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] + 6 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \right. \\
& \quad \#1^2 - 3 i \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^2 + 6 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^4 - \\
& \quad 3 i \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^4 + 2 \text{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1} \right] \#1^6 - \\
& \quad i \log\left[1 - 2 \cos[c+d x] \#1 + \#1^2 \right] \#1^6 \Big) \Big/ (-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7) \&
\end{aligned}$$

Problem 408: Result is not expressed in closed-form.

$$\int \frac{\sec[c+d x]}{a - b \sin[c+d x]^4} dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} + \sqrt{b}) d} + \frac{\operatorname{ArcTanh}[\sin[c+d x]]}{(a-b) d} - \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} - \sqrt{b}) d}$$

Result (type 7, 342 leaves):

$$\frac{1}{8 a d - 8 b d} \left(-8 \operatorname{Log}[\cos[\frac{1}{2} (c+d x)] - \sin[\frac{1}{2} (c+d x)]] + 8 \operatorname{Log}[\cos[\frac{1}{2} (c+d x)] + \sin[\frac{1}{2} (c+d x)]] - b \operatorname{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \left(2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - i \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] - 10 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + 5 i \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - 10 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 + 5 i \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + 2 \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - i \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) / (-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7) \&] \right)$$

Problem 409: Result is not expressed in closed-form.

$$\int \frac{\sec[c+d x]^3}{a - b \sin[c+d x]^4} dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$\frac{b^{3/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} + \sqrt{b})^2 d} + \frac{(a - 5 b) \operatorname{ArcTanh}[\sin[c+d x]]}{2 (a-b)^2 d} + \frac{b^{3/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c+d x]}{a^{1/4}}\right]}{2 a^{3/4} (\sqrt{a} - \sqrt{b})^2 d} + \frac{1}{4 (a-b) d (1 - \sin[c+d x])} - \frac{1}{4 (a-b) d (1 + \sin[c+d x])}$$

Result (type 7, 529 leaves):

$$\begin{aligned}
& \frac{1}{4 (a - b)^2 d} \left(-2 (a - 5 b) \operatorname{Log}[\cos[\frac{1}{2} (c + d x)] - \sin[\frac{1}{2} (c + d x)]] + \right. \\
& 2 (a - 5 b) \operatorname{Log}[\cos[\frac{1}{2} (c + d x)] + \sin[\frac{1}{2} (c + d x)]] + \\
& b \operatorname{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7}] \\
& \left(2 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] - i b \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] - \right. \\
& 4 a \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^2 - 6 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^2 + \\
& 2 i a \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^2 + 3 i b \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^2 - \\
& 4 a \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 - 6 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^4 + \\
& 2 i a \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^4 + 3 i b \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^4 + \\
& \left. 2 b \operatorname{ArcTan}\left[\frac{\sin[c + d x]}{\cos[c + d x] - \#1}\right] \#1^6 - i b \operatorname{Log}[1 - 2 \cos[c + d x] \#1 + \#1^2] \#1^6 \right) \&] + \\
& \frac{a - b}{(\cos[\frac{1}{2} (c + d x)] - \sin[\frac{1}{2} (c + d x)])^2} + \frac{-a + b}{(\cos[\frac{1}{2} (c + d x)] + \sin[\frac{1}{2} (c + d x)])^2} \Bigg)
\end{aligned}$$

Problem 410: Result is not expressed in closed-form.

$$\int \frac{\sec[c + d x]^5}{a - b \sin[c + d x]^4} dx$$

Optimal (type 3, 249 leaves, 7 steps) :

$$\begin{aligned}
& \frac{b^{5/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \sin[c + d x]}{a^{1/4}}\right]}{2 a^{3/4} \left(\sqrt{a} + \sqrt{b}\right)^3 d} + \frac{(3 a^2 - 6 a b + 35 b^2) \operatorname{ArcTanh}[\sin[c + d x]]}{8 (a - b)^3 d} - \\
& \frac{b^{5/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \sin[c + d x]}{a^{1/4}}\right]}{2 a^{3/4} \left(\sqrt{a} - \sqrt{b}\right)^3 d} + \frac{1}{16 (a - b) d (1 - \sin[c + d x])^2} + \frac{3 a - 11 b}{16 (a - b)^2 d (1 - \sin[c + d x])} - \\
& \frac{1}{16 (a - b) d (1 + \sin[c + d x])^2} - \frac{3 a - 11 b}{16 (a - b)^2 d (1 + \sin[c + d x])}
\end{aligned}$$

Result (type 7, 731 leaves) :

$$\begin{aligned}
& \frac{1}{16 (a-b)^3 d} \left(-2 (3 a^2 - 6 a b + 35 b^2) \operatorname{Log}[\cos[\frac{1}{2} (c+d x)] - \sin[\frac{1}{2} (c+d x)]] + \right. \\
& 2 (3 a^2 - 6 a b + 35 b^2) \operatorname{Log}[\cos[\frac{1}{2} (c+d x)] + \sin[\frac{1}{2} (c+d x)]] - \\
& 2 b^2 \operatorname{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\
& \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(2 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] + \right. \\
& 6 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] - i a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] - \\
& 3 i b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] - 26 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 - \\
& 14 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^2 + 13 i a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 + \\
& 7 i b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^2 - 26 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 - \\
& 14 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^4 + 13 i a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + \\
& 7 i b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^4 + 2 a \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 + \\
& 6 b \operatorname{ArcTan}\left[\frac{\sin[c+d x]}{\cos[c+d x] - \#1}\right] \#1^6 - i a \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 - \\
& \left. 3 i b \operatorname{Log}[1 - 2 \cos[c+d x] \#1 + \#1^2] \#1^6 \right) \&] + \\
& \frac{(a-b)^2}{(\cos[\frac{1}{2} (c+d x)] - \sin[\frac{1}{2} (c+d x)])^4} + \frac{(3 a - 11 b) (a-b)}{(\cos[\frac{1}{2} (c+d x)] - \sin[\frac{1}{2} (c+d x)])^2} - \\
& \frac{(a-b)^2}{(\cos[\frac{1}{2} (c+d x)] + \sin[\frac{1}{2} (c+d x)])^4} + \\
& \left. \frac{(a-b) (-3 a + 11 b)}{(\cos[\frac{1}{2} (c+d x)] + \sin[\frac{1}{2} (c+d x)])^2} \right)
\end{aligned}$$

Problem 420: Unable to integrate problem.

$$\int \cos[e + f x]^5 (a + b \sin[e + f x]^4)^p dx$$

Optimal (type 5, 197 leaves, 8 steps):

$$\frac{\sin[e+fx] (a+b \sin[e+fx]^4)^{1+p}}{b f (5+4 p)} - \frac{1}{b f (5+4 p)} \\ (a-b (5+4 p)) \text{Hypergeometric2F1}\left[\frac{1}{4}, -p, \frac{5}{4}, -\frac{b \sin[e+fx]^4}{a}\right] \sin[e+fx] (a+b \sin[e+fx]^4)^p \\ \left(1 + \frac{b \sin[e+fx]^4}{a}\right)^{-p} - \frac{1}{3 f} 2 \text{Hypergeometric2F1}\left[\frac{3}{4}, -p, \frac{7}{4}, -\frac{b \sin[e+fx]^4}{a}\right] \\ \sin[e+fx]^3 (a+b \sin[e+fx]^4)^p \left(1 + \frac{b \sin[e+fx]^4}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \cos[e+fx]^5 (a+b \sin[e+fx]^4)^p dx$$

Problem 421: Unable to integrate problem.

$$\int \cos[e+fx]^3 (a+b \sin[e+fx]^4)^p dx$$

Optimal (type 5, 140 leaves, 7 steps):

$$\frac{1}{f} \text{Hypergeometric2F1}\left[\frac{1}{4}, -p, \frac{5}{4}, -\frac{b \sin[e+fx]^4}{a}\right] \sin[e+fx] (a+b \sin[e+fx]^4)^p \\ \left(1 + \frac{b \sin[e+fx]^4}{a}\right)^{-p} - \frac{1}{3 f} \text{Hypergeometric2F1}\left[\frac{3}{4}, -p, \frac{7}{4}, -\frac{b \sin[e+fx]^4}{a}\right] \\ \sin[e+fx]^3 (a+b \sin[e+fx]^4)^p \left(1 + \frac{b \sin[e+fx]^4}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \cos[e+fx]^3 (a+b \sin[e+fx]^4)^p dx$$

Problem 423: Unable to integrate problem.

$$\int \sec[e+fx] (a+b \sin[e+fx]^4)^p dx$$

Optimal (type 6, 158 leaves, 7 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{4}, 1, -p, \frac{5}{4}, \sin[e+fx]^4, -\frac{b \sin[e+fx]^4}{a}\right] \sin[e+fx] (a+b \sin[e+fx]^4)^p \\ \left(1 + \frac{b \sin[e+fx]^4}{a}\right)^{-p} + \frac{1}{3 f} \text{AppellF1}\left[\frac{3}{4}, 1, -p, \frac{7}{4}, \sin[e+fx]^4, -\frac{b \sin[e+fx]^4}{a}\right] \\ \sin[e+fx]^3 (a+b \sin[e+fx]^4)^p \left(1 + \frac{b \sin[e+fx]^4}{a}\right)^{-p}$$

Result (type 8, 23 leaves):

$$\int \sec[e+fx] (a+b \sin[e+fx]^4)^p dx$$

Problem 424: Unable to integrate problem.

$$\int \sec [e + f x]^3 (a + b \sin [e + f x]^4)^p dx$$

Optimal (type 6, 239 leaves, 9 steps):

$$\begin{aligned} & \frac{1}{f} \text{AppellF1}\left[\frac{1}{4}, 2, -p, \frac{5}{4}, \sin[e + f x]^4, -\frac{b \sin[e + f x]^4}{a}\right] \\ & \quad \sin[e + f x] (a + b \sin[e + f x]^4)^p \left(1 + \frac{b \sin[e + f x]^4}{a}\right)^{-p} + \frac{1}{3f} \\ & 2 \text{AppellF1}\left[\frac{3}{4}, 2, -p, \frac{7}{4}, \sin[e + f x]^4, -\frac{b \sin[e + f x]^4}{a}\right] \sin[e + f x]^3 (a + b \sin[e + f x]^4)^p \\ & \quad \left(1 + \frac{b \sin[e + f x]^4}{a}\right)^{-p} + \frac{1}{5f} \text{AppellF1}\left[\frac{5}{4}, 2, -p, \frac{9}{4}, \sin[e + f x]^4, -\frac{b \sin[e + f x]^4}{a}\right] \\ & \quad \sin[e + f x]^5 (a + b \sin[e + f x]^4)^p \left(1 + \frac{b \sin[e + f x]^4}{a}\right)^{-p} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \sec [e + f x]^3 (a + b \sin [e + f x]^4)^p dx$$

Problem 431: Unable to integrate problem.

$$\int \cos [e + f x]^5 (a + b \sin [e + f x]^n)^p dx$$

Optimal (type 5, 226 leaves, 9 steps):

$$\begin{aligned} & \frac{1}{f} \text{Hypergeometric2F1}\left[\frac{1}{n}, -p, 1 + \frac{1}{n}, -\frac{b \sin[e + f x]^n}{a}\right] \\ & \quad \sin[e + f x] (a + b \sin[e + f x]^n)^p \left(1 + \frac{b \sin[e + f x]^n}{a}\right)^{-p} - \frac{1}{3f} \\ & 2 \text{Hypergeometric2F1}\left[\frac{3}{n}, -p, \frac{3+n}{n}, -\frac{b \sin[e + f x]^n}{a}\right] \sin[e + f x]^3 (a + b \sin[e + f x]^n)^p \\ & \quad \left(1 + \frac{b \sin[e + f x]^n}{a}\right)^{-p} + \frac{1}{5f} \text{Hypergeometric2F1}\left[\frac{5}{n}, -p, \frac{5+n}{n}, -\frac{b \sin[e + f x]^n}{a}\right] \\ & \quad \sin[e + f x]^5 (a + b \sin[e + f x]^n)^p \left(1 + \frac{b \sin[e + f x]^n}{a}\right)^{-p} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \cos [e + f x]^5 (a + b \sin [e + f x]^n)^p dx$$

Problem 432: Unable to integrate problem.

$$\int \cos [e + f x]^3 (a + b \sin [e + f x]^n)^p dx$$

Optimal (type 5, 148 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{f} \text{Hypergeometric2F1}\left[\frac{1}{n}, -p, 1 + \frac{1}{n}, -\frac{b \sin[e + fx]^n}{a}\right] \sin[e + fx] (a + b \sin[e + fx]^n)^p \\ & \left(1 + \frac{b \sin[e + fx]^n}{a}\right)^{-p} - \frac{1}{3f} \text{Hypergeometric2F1}\left[\frac{3}{n}, -p, \frac{3+n}{n}, -\frac{b \sin[e + fx]^n}{a}\right] \\ & \sin[e + fx]^3 (a + b \sin[e + fx]^n)^p \left(1 + \frac{b \sin[e + fx]^n}{a}\right)^{-p} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \cos[e + fx]^3 (a + b \sin[e + fx]^n)^p dx$$

Problem 474: Result more than twice size of optimal antiderivative.

$$\int \frac{\tan[e + fx]^2}{\sqrt{a - a \sin[e + fx]^2}} dx$$

Optimal (type 3, 62 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[\sin[e + fx]] \cos[e + fx]}{2f \sqrt{a \cos[e + fx]^2}} + \frac{\tan[e + fx]}{2f \sqrt{a \cos[e + fx]^2}}$$

Result (type 3, 142 leaves):

$$\begin{aligned} & \left(\sec[e + fx] \left(\log[\cos[\frac{1}{2}(e + fx)]] - \sin[\frac{1}{2}(e + fx)] \right] + \cos[2(e + fx)] \right. \\ & \left(\log[\cos[\frac{1}{2}(e + fx)]] - \sin[\frac{1}{2}(e + fx)] \right] - \log[\cos[\frac{1}{2}(e + fx)]] + \sin[\frac{1}{2}(e + fx)] \right) - \\ & \left. \log[\cos[\frac{1}{2}(e + fx)]] + \sin[\frac{1}{2}(e + fx)] \right] + 2 \sin[e + fx] \Bigg) / \left(4f \sqrt{a \cos[e + fx]^2} \right) \end{aligned}$$

Problem 483: Result more than twice size of optimal antiderivative.

$$\int \frac{\tan[e + fx]^2}{(a - a \sin[e + fx]^2)^{3/2}} dx$$

Optimal (type 3, 106 leaves, 5 steps):

$$-\frac{\text{ArcTanh}[\sin[e + fx]] \cos[e + fx]}{8af \sqrt{a \cos[e + fx]^2}} - \frac{\tan[e + fx]}{8af \sqrt{a \cos[e + fx]^2}} + \frac{\sec[e + fx]^2 \tan[e + fx]}{4af \sqrt{a \cos[e + fx]^2}}$$

Result (type 3, 213 leaves):

$$\begin{aligned} & \frac{1}{64 f (a \cos[e+f x]^2)^{3/2}} \sec[e+f x] \left(3 \log[\cos[\frac{1}{2} (e+f x)]] - \sin[\frac{1}{2} (e+f x)] \right) + \\ & 4 \cos[2 (e+f x)] \left(\log[\cos[\frac{1}{2} (e+f x)]] - \sin[\frac{1}{2} (e+f x)] \right) - \\ & \log[\cos[\frac{1}{2} (e+f x)]] + \sin[\frac{1}{2} (e+f x)] \right) + \cos[4 (e+f x)] \\ & \left(\log[\cos[\frac{1}{2} (e+f x)]] - \sin[\frac{1}{2} (e+f x)] \right) - \log[\cos[\frac{1}{2} (e+f x)]] + \sin[\frac{1}{2} (e+f x)] \Big) - \\ & 3 \log[\cos[\frac{1}{2} (e+f x)]] + \sin[\frac{1}{2} (e+f x)] \Big] + 14 \sin[e+f x] - 2 \sin[3 (e+f x)] \Big) \end{aligned}$$

Problem 543: Result more than twice size of optimal antiderivative.

$$\int (a + b \sin[e+f x]^2)^p (d \tan[e+f x])^m dx$$

Optimal (type 6, 120 leaves, 3 steps):

$$\begin{aligned} & \frac{1}{d f (1+m)} \text{AppellF1}[\frac{1+m}{2}, \frac{1+m}{2}, -p, \frac{3+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}] \\ & (\cos[e+f x]^2)^{\frac{1-m}{2}} (a + b \sin[e+f x]^2)^p \left(1 + \frac{b \sin[e+f x]^2}{a} \right)^{-p} (d \tan[e+f x])^{1+m} \end{aligned}$$

Result (type 6, 260 leaves):

$$\begin{aligned} & \left(a (3+m) \text{AppellF1}[\frac{1+m}{2}, \frac{1+m}{2}, -p, \frac{3+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}] \right. \\ & \left. (a + b \sin[e+f x]^2)^p \tan[e+f x] (d \tan[e+f x])^m \right) / \\ & \left(f (1+m) \left(a (3+m) \text{AppellF1}[\frac{1+m}{2}, \frac{1+m}{2}, -p, \frac{3+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}] + \right. \right. \\ & \left. \left. 2 b p \text{AppellF1}[\frac{3+m}{2}, \frac{1+m}{2}, 1-p, \frac{5+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}] + a (1+m) \right. \right. \\ & \left. \left. \text{AppellF1}[\frac{3+m}{2}, \frac{3+m}{2}, -p, \frac{5+m}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}] \right) \sin[e+f x]^2 \right) \end{aligned}$$

Problem 547: Unable to integrate problem.

$$\int \cot[c+d x]^3 (a + b \sin[c+d x]^2)^p dx$$

Optimal (type 5, 95 leaves, 3 steps):

$$\begin{aligned} & - \frac{\csc[c+d x]^2 (a + b \sin[c+d x]^2)^{1+p}}{2 a d} + \frac{1}{2 a^2 d (1+p)} \\ & (a - b p) \text{Hypergeometric2F1}[1, 1+p, 2+p, 1 + \frac{b \sin[c+d x]^2}{a}] (a + b \sin[c+d x]^2)^{1+p} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\cot[c + d x]^3}{a + b \sin[x]^3} (a + b \sin[c + d x]^2)^p dx$$

Problem 552: Result is not expressed in closed-form.

$$\int \frac{\cot[x]^3}{a + b \sin[x]^3} dx$$

Optimal (type 3, 153 leaves, 11 steps):

$$\begin{aligned} & \frac{b^{2/3} \operatorname{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \sin[x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{5/3}}-\frac{\csc [x]^2}{2 a}-\frac{\log [\sin [x]]}{a}-\frac{b^{2/3} \log \left[a^{1/3}+b^{1/3} \sin [x]\right]}{3 a^{5/3}}+ \\ & \frac{b^{2/3} \log \left[a^{2/3}-a^{1/3} b^{1/3} \sin [x]+b^{2/3} \sin [x]^2\right]}{6 a^{5/3}}+\frac{\log \left[a+b \sin [x]^3\right]}{3 a} \end{aligned}$$

Result (type 7, 210 leaves):

$$\begin{aligned} & \frac{1}{24 a} \\ & \left(8 \operatorname{RootSum}\left[a+3 a \#1^2+8 b \#1^3+3 a \#1^4+a \#1^6 \&, \left(-b \log \left[-\#1+\tan \left[\frac{x}{2}\right]\right]+a \log \left[-\#1+\tan \left[\frac{x}{2}\right]\right] \#1+\right.\right. \\ & 4 b \log \left[-\#1+\tan \left[\frac{x}{2}\right]\right] \#1^2+2 a \log \left[-\#1+\tan \left[\frac{x}{2}\right]\right] \#1^3+b \log \left[-\#1+\tan \left[\frac{x}{2}\right]\right] \#1^4+ \\ & \left.\left.a \log \left[-\#1+\tan \left[\frac{x}{2}\right]\right] \#1^5\right) /\left(a \#1+4 b \#1^2+2 a \#1^3+a \#1^5\right) \&\right]- \\ & 3 \left(\csc \left[\frac{x}{2}\right]^2+8 \left(\log \left[\sec \left[\frac{x}{2}\right]^2\right]+\log [\sin [x]]\right)+\sec \left[\frac{x}{2}\right]^2\right) \end{aligned}$$

Problem 554: Result more than twice size of optimal antiderivative.

$$\int \frac{\cot[x]}{\sqrt{a+b \sin[x]^3}} dx$$

Optimal (type 3, 28 leaves, 4 steps):

$$-\frac{2 \operatorname{Arctanh}\left[\frac{\sqrt{a+b \sin [x]^3}}{\sqrt{a}}\right]}{3 \sqrt{a}}$$

Result (type 3, 66 leaves):

$$-\frac{2 \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \csc [x]^{3/2}}{\sqrt{b}}\right] \sqrt{\frac{b+a \csc [x]^3}{b}}}{3 \sqrt{a} \csc [x]^{3/2} \sqrt{a+b \sin [x]^3}}$$

Problem 555: Result more than twice size of optimal antiderivative.

$$\int \cot[c + dx] \sqrt{a + b \sin[c + dx]^4} \, dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \sin [c+d x]^4}{\sqrt{a}}\right]}{2 d}+\frac{\sqrt{a+b} \sin [c+d x]^4}{2 d}$$

Result (type 3, 166 leaves):

$$\begin{aligned} & \left(\sqrt{\cos [c+d x]^4 \left(a+2 a \tan [c+d x]^2+(a+b) \tan [c+d x]^4\right)}\right. \\ & \left.\left(\sqrt{a} \left(\log [\tan [c+d x]^2]-\log \left[a+a \tan [c+d x]^2+\sqrt{a} \sqrt{a \sec [c+d x]^4+b \tan [c+d x]^4}\right]\right)\right.\right. \\ & \left.\left.\left.\sec [c+d x]^2+\sqrt{a \sec [c+d x]^4+b \tan [c+d x]^4}\right)\right)\right) \Big/ \left(2 d \sqrt{a \sec [c+d x]^4+b \tan [c+d x]^4}\right) \end{aligned}$$

Problem 556: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\tan [c+d x]^3}{\sqrt{a+b \sin [c+d x]^4}} \, dx$$

Optimal (type 3, 89 leaves, 4 steps):

$$-\frac{a \operatorname{ArcTanh}\left[\frac{a+b \sin [c+d x]^2}{\sqrt{a+b} \sqrt{a+b \sin [c+d x]^4}}\right]}{2 (a+b)^{3/2} d}+\frac{\sec [c+d x]^2 \sqrt{a+b \sin [c+d x]^4}}{2 (a+b) d}$$

Result (type 4, 63 448 leaves): Display of huge result suppressed!

Problem 557: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\tan [c+d x]}{\sqrt{a+b \sin [c+d x]^4}} \, dx$$

Optimal (type 3, 51 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{a+b \sin [c+d x]^2}{\sqrt{a+b} \sqrt{a+b \sin [c+d x]^4}}\right]}{2 \sqrt{a+b} d}$$

Result (type 4, 39 909 leaves): Display of huge result suppressed!

Problem 558: Result more than twice size of optimal antiderivative.

$$\int \frac{\cot[c + dx]}{\sqrt{a + b \sin[c + dx]^4}} dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sin[c+d x]^4}}{\sqrt{a}}\right]}{2 \sqrt{a} d}$$

Result (type 3, 142 leaves):

$$\begin{aligned} & \left(\sqrt{8 a + 3 b - 4 b \cos[2(c + d x)]} + b \cos[4(c + d x)] \right) \\ & \left(\operatorname{Log}[\tan[c + d x]^2] - \operatorname{Log}[a + a \tan[c + d x]^2 + \sqrt{a} \sqrt{a \sec[c + d x]^4 + b \tan[c + d x]^4}] \right) \\ & \sec[c + d x]^2 \Big/ \left(4 \sqrt{2} \sqrt{a} d \sqrt{a + 2 a \tan[c + d x]^2 + (a + b) \tan[c + d x]^4} \right) \end{aligned}$$

Problem 559: Result more than twice size of optimal antiderivative.

$$\int \frac{\cot[c + dx]^3}{\sqrt{a + b \sin[c + dx]^4}} dx$$

Optimal (type 3, 70 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sin[c+d x]^4}}{\sqrt{a}}\right]}{2 \sqrt{a} d} - \frac{\csc[c + d x]^2 \sqrt{a + b \sin[c + d x]^4}}{2 a d}$$

Result (type 3, 185 leaves):

$$\begin{aligned} & - \left(\left(\sqrt{8 a + 3 b - 4 b \cos[2(c + d x)]} + b \cos[4(c + d x)] \right) \right. \\ & \left. \left(\sqrt{a} \left(\operatorname{Log}[\tan[c + d x]^2] - \operatorname{Log}[a + a \tan[c + d x]^2 + \sqrt{a} \sqrt{a \sec[c + d x]^4 + b \tan[c + d x]^4}] \right) \right. \right. \\ & \left. \left. \sec[c + d x]^2 + \csc[c + d x]^2 \sqrt{a + 2 a \tan[c + d x]^2 + (a + b) \tan[c + d x]^4} \right) \right) \Big/ \\ & \left(4 \sqrt{2} a d \sqrt{a + 2 a \tan[c + d x]^2 + (a + b) \tan[c + d x]^4} \right) \end{aligned}$$

Problem 561: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\tan[c + dx]^2}{\sqrt{a + b \sin[c + dx]^4}} dx$$

Optimal (type 4, 411 leaves, 4 steps):

$$\begin{aligned}
& \frac{\cos[c+dx] \sin[c+dx] \left(a + 2a \tan[c+dx]^2 + (a+b) \tan[c+dx]^4\right)}{\sqrt{a+b} d \sqrt{a+b \sin[c+dx]^4} \left(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2\right)} - \\
& \left(a^{1/4} \cos[c+dx]^2 \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{(a+b)^{1/4} \tan[c+dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a+b}}\right)\right] \right. \\
& \left. \left(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2 \right) \sqrt{\frac{a + 2a \tan[c+dx]^2 + (a+b) \tan[c+dx]^4}{\left(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2\right)^2}} \right) / \\
& \left((a+b)^{3/4} d \sqrt{a+b \sin[c+dx]^4} \right) + \left(a^{1/4} \cos[c+dx]^2 \right. \\
& \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{(a+b)^{1/4} \tan[c+dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a+b}}\right)\right] \left(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2\right) \right. \\
& \left. \sqrt{\frac{a + 2a \tan[c+dx]^2 + (a+b) \tan[c+dx]^4}{\left(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2\right)^2}} \right) / \left(2 (a+b)^{3/4} d \sqrt{a+b \sin[c+dx]^4} \right)
\end{aligned}$$

Result (type 4, 291 leaves):

$$\begin{aligned}
& - \left(\left(2 \pm \sqrt{2} \sqrt{a} \cos[c+dx]^2 \left(\text{EllipticE}\left[\pm \text{ArcSinh}\left[\sqrt{1 - \frac{\pm \sqrt{b}}{\sqrt{a}}} \tan[c+dx]\right], \frac{\sqrt{a} \pm \pm \sqrt{b}}{\sqrt{a} \mp \pm \sqrt{b}}\right] - \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\pm \text{ArcSinh}\left[\sqrt{1 - \frac{\pm \sqrt{b}}{\sqrt{a}}} \tan[c+dx]\right], \frac{\sqrt{a} \pm \pm \sqrt{b}}{\sqrt{a} \mp \pm \sqrt{b}}\right]\right) \right. \\
& \left. \sqrt{1 + \left(1 - \frac{\pm \sqrt{b}}{\sqrt{a}}\right) \tan[c+dx]^2} \sqrt{1 + \left(1 + \frac{\pm \sqrt{b}}{\sqrt{a}}\right) \tan[c+dx]^2} \right) / \\
& \left. \left(\left(\sqrt{a} \pm \pm \sqrt{b} \right) \sqrt{1 - \frac{\pm \sqrt{b}}{\sqrt{a}}} d \sqrt{8a + 3b - 4b \cos[2(c+dx)] + b \cos[4(c+dx)]} \right) \right)
\end{aligned}$$

Problem 562: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+b \sin[c+dx]^4}} dx$$

Optimal (type 4, 162 leaves, 2 steps):

$$\left(\cos[c+dx]^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{(a+b)^{1/4} \tan[c+dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a+b}}\right)\right] \right. \\ \left. \left(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2 \right) \sqrt{\frac{a + 2 a \tan[c+dx]^2 + (a+b) \tan[c+dx]^4}{\left(\sqrt{a} + \sqrt{a+b} \tan[c+dx]^2\right)^2}} \right) / \\ \left(2 a^{1/4} (a+b)^{1/4} d \sqrt{a+b \sin[c+dx]^4} \right)$$

Result (type 4, 195 leaves) :

$$- \left(\left(2 i \cos[c+dx]^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \tan[c+dx]\right], \frac{\sqrt{a} + i \sqrt{b}}{\sqrt{a} - i \sqrt{b}}\right] \right. \right. \\ \left. \left. \sqrt{1 + \left(1 + \frac{i \sqrt{b}}{\sqrt{a}}\right) \tan[c+dx]^2} \sqrt{2 + \left(2 - \frac{2 i \sqrt{b}}{\sqrt{a}}\right) \tan[c+dx]^2} \right) / \right. \\ \left. \left(\sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} d \sqrt{8 a + 3 b - 4 b \cos[2 (c+dx)] + b \cos[4 (c+dx)]} \right) \right)$$

Problem 563: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cot[c+dx]^2}{\sqrt{a+b \sin[c+dx]^4}} dx$$

Optimal (type 4, 477 leaves, 6 steps) :

$$\begin{aligned}
& - \left((\cos[c + dx]^2 \cot[c + dx] (a + 2 a \tan[c + dx]^2 + (a + b) \tan[c + dx]^4)) \right) / \\
& \quad \left(a d \sqrt{a + b \sin[c + dx]^4} \right) + \\
& \left(\sqrt{a + b} \cos[c + dx] \sin[c + dx] (a + 2 a \tan[c + dx]^2 + (a + b) \tan[c + dx]^4) \right) / \\
& \quad \left(a d \sqrt{a + b \sin[c + dx]^4} (\sqrt{a} + \sqrt{a + b} \tan[c + dx]^2) \right) - \\
& \left((a + b)^{1/4} \cos[c + dx]^2 \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{(a + b)^{1/4} \tan[c + dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a + b}}\right)\right]\right. \\
& \quad \left. \left(\sqrt{a} + \sqrt{a + b} \tan[c + dx]^2\right) \sqrt{\frac{a + 2 a \tan[c + dx]^2 + (a + b) \tan[c + dx]^4}{(\sqrt{a} + \sqrt{a + b} \tan[c + dx]^2)^2}}\right) / \\
& \quad \left(a^{3/4} d \sqrt{a + b \sin[c + dx]^4} \right) + \left((a + b)^{1/4} \cos[c + dx]^2 \right. \\
& \quad \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{(a + b)^{1/4} \tan[c + dx]}{a^{1/4}}\right], \frac{1}{2} \left(1 - \frac{\sqrt{a}}{\sqrt{a + b}}\right)\right] (\sqrt{a} + \sqrt{a + b} \tan[c + dx]^2) \right. \\
& \quad \left. \sqrt{\frac{a + 2 a \tan[c + dx]^2 + (a + b) \tan[c + dx]^4}{(\sqrt{a} + \sqrt{a + b} \tan[c + dx]^2)^2}}\right) / \left(2 a^{3/4} d \sqrt{a + b \sin[c + dx]^4} \right)
\end{aligned}$$

Result (type 4, 378 leaves):

$$\begin{aligned}
& - \frac{\sqrt{8a + 3b - 4b \cos[2(c + dx)] + b \cos[4(c + dx)]} \cot[c + dx]}{2\sqrt{2}ad} - \\
& \left(\cos[c + dx]^4 \left(a \sec[c + dx]^4 \tan[c + dx] + b \tan[c + dx]^5 + \frac{1}{\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}}}} \right) \right. \\
& \left(\frac{i}{2}a + \sqrt{a} - \sqrt{b} \right) \left(\text{EllipticE}\left[\frac{i}{2} \text{ArcSinh}\left[\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}}}\right] \tan[c + dx], \frac{\sqrt{a} + i\sqrt{b}}{\sqrt{a} - i\sqrt{b}}\right] - \right. \\
& \left. \text{EllipticF}\left[\frac{i}{2} \text{ArcSinh}\left[\sqrt{1 - \frac{i\sqrt{b}}{\sqrt{a}}}\right] \tan[c + dx], \frac{\sqrt{a} + i\sqrt{b}}{\sqrt{a} - i\sqrt{b}}\right] \right) \\
& \left. \sec[c + dx]^2 \sqrt{1 + \left(1 - \frac{i\sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} \sqrt{1 + \left(1 + \frac{i\sqrt{b}}{\sqrt{a}}\right) \tan[c + dx]^2} \right) / \\
& \left(a d \sqrt{\cos[c + dx]^4 (a + 2a \tan[c + dx]^2 + (a + b) \tan[c + dx]^4)} \right)
\end{aligned}$$

Problem 565: Result more than twice size of optimal antiderivative.

$$\int (a + b \sin[c + dx]^4)^p \tan[c + dx]^3 dx$$

Optimal (type 6, 279 leaves, 11 steps):

$$\begin{aligned}
& - \left(\left((a + b + 2bp) \text{Hypergeometric2F1}[1, 1+p, 2+p, \frac{a + b \sin[c + dx]^4}{a + b}] (a + b \sin[c + dx]^4)^{1+p} \right) / \right. \\
& \left. \left(4(a + b)^2 d (1 + p) \right) \right) + \frac{\sec[c + dx]^2 (a + b \sin[c + dx]^4)^{1+p}}{2(a + b)d} - \\
& \frac{1}{2(a + b)d} (a + b + 2bp) \text{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, \sin[c + dx]^4, -\frac{b \sin[c + dx]^4}{a}\right] \\
& \sin[c + dx]^2 (a + b \sin[c + dx]^4)^p \left(1 + \frac{b \sin[c + dx]^4}{a}\right)^{-p} + \\
& \frac{1}{2(a + b)d} b (1 + 2p) \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \sin[c + dx]^4}{a}\right] \\
& \sin[c + dx]^2 (a + b \sin[c + dx]^4)^p \left(1 + \frac{b \sin[c + dx]^4}{a}\right)^{-p}
\end{aligned}$$

Result (type 6, 2007 leaves):

$$\begin{aligned}
& - \left[\left(\left((1-2p) \text{AppellF1}[-2p, -p, -p, 1-2p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}] + \right. \right. \right. \\
& \quad 2p \text{AppellF1}[1-2p, -p, -p, 2-2p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}] \\
& \quad \left. \left. \left. \sec[c+d x]^2 \right) (a+b \sin[c+d x]^4)^p \tan[c+d x]^3 \right. \right. \\
& \quad \left(\frac{-a + \sqrt{-a b} - (a+b) \tan[c+d x]^2}{b + \sqrt{-a b}} \right)^{-p} \left(\frac{a + \sqrt{-a b} + (a+b) \tan[c+d x]^2}{-b + \sqrt{-a b}} \right)^{-p} \\
& \quad \left. \left. \left. (\cos[c+d x]^4 (a+2a \tan[c+d x]^2 + (a+b) \tan[c+d x]^4))^p \right) \right] / \\
& \left(4dp (-1+2p) \left(\frac{1}{2(-b + \sqrt{-a b}) (-1+2p)} (a+b) \sec[c+d x]^2 \left((1-2p) \text{AppellF1}[-2p, -p, \right. \right. \right. \\
& \quad \left. \left. \left. -p, 1-2p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}] + 2p \text{AppellF1}[1-2p, -p, \right. \right. \right. \\
& \quad \left. \left. \left. -p, 2-2p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}] \sec[c+d x]^2 \right) \tan[c+d x] \right. \right. \\
& \quad \left(\frac{-a + \sqrt{-a b} - (a+b) \tan[c+d x]^2}{b + \sqrt{-a b}} \right)^{-p} \left(\frac{a + \sqrt{-a b} + (a+b) \tan[c+d x]^2}{-b + \sqrt{-a b}} \right)^{-1-p} \\
& \quad \left. \left. \left. (\cos[c+d x]^4 (a+2a \tan[c+d x]^2 + (a+b) \tan[c+d x]^4))^p - \right. \right. \right. \\
& \quad \frac{1}{2(b + \sqrt{-a b}) (-1+2p)} (a+b) \sec[c+d x]^2 \left((1-2p) \text{AppellF1}[-2p, -p, -p, \right. \right. \right. \\
& \quad \left. \left. \left. 1-2p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}] + 2p \text{AppellF1}[1-2p, -p, -p, \right. \right. \right. \\
& \quad \left. \left. \left. 2-2p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}] \sec[c+d x]^2 \right) \tan[c+d x] \right. \right. \\
& \quad \left(\frac{-a + \sqrt{-a b} - (a+b) \tan[c+d x]^2}{b + \sqrt{-a b}} \right)^{-1-p} \left(\frac{a + \sqrt{-a b} + (a+b) \tan[c+d x]^2}{-b + \sqrt{-a b}} \right)^{-p} \\
& \quad \left. \left. \left. (\cos[c+d x]^4 (a+2a \tan[c+d x]^2 + (a+b) \tan[c+d x]^4))^p - \right. \right. \right. \\
& \quad \frac{1}{4p (-1+2p)} \left(\frac{-a + \sqrt{-a b} - (a+b) \tan[c+d x]^2}{b + \sqrt{-a b}} \right)^{-p} \left(\frac{a + \sqrt{-a b} + (a+b) \tan[c+d x]^2}{-b + \sqrt{-a b}} \right)^{-p} \\
& \quad \left. \left. \left. (\cos[c+d x]^4 (a+2a \tan[c+d x]^2 + (a+b) \tan[c+d x]^4))^p \right) \right] \\
& \quad \left(4p \text{AppellF1}[1-2p, -p, -p, 2-2p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}] \right. \\
& \quad \left. \sec[c+d x]^2 \tan[c+d x] + (1-2p) \left(- \left(\left(4(a+b) p^2 \text{AppellF1}[1-2p, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. -p, 1-2p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}] + 2p \text{AppellF1}[1-2p, -p, -p, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. 1-2p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}] \sec[c+d x]^2 \right) \tan[c+d x] \right) \right. \right. \right. \right. \right. \right. \\
& \quad \left(\frac{-a + \sqrt{-a b} - (a+b) \tan[c+d x]^2}{b + \sqrt{-a b}} \right)^{-p} \left(\frac{a + \sqrt{-a b} + (a+b) \tan[c+d x]^2}{-b + \sqrt{-a b}} \right)^{-p} \\
& \quad \left. \left. \left. (\cos[c+d x]^4 (a+2a \tan[c+d x]^2 + (a+b) \tan[c+d x]^4))^p \right) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& 1 - p, -p, 2 - 2p, - \frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b + \sqrt{-a b}} \\
& \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \Bigg) \Bigg/ \left(\left(-b + \sqrt{-a b} \right) (1 - 2p) \right) + \\
& \left(4 (a+b) p^2 \operatorname{AppellF1}[1 - 2p, -p, 1 - p, 2 - 2p, - \frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b + \sqrt{-a b}}, \right. \\
& \left. \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b + \sqrt{-a b}}] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \right) \Bigg/ \left(\left(b + \sqrt{-a b} \right) (1 - 2p) \right) + \\
& 2p \operatorname{Sec}[c+d x]^2 \left(\left(2 (a+b) (1 - 2p) p \operatorname{AppellF1}[2 - 2p, 1 - p, -p, 3 - 2p, \right. \right. \\
& \left. \left. - \frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b + \sqrt{-a b}}] \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \right) \Bigg/ \right. \\
& \left. \left(-b + \sqrt{-a b} \right) (2 - 2p) \right) - \left(2 (a+b) (1 - 2p) p \operatorname{AppellF1}[2 - 2p, \right. \\
& \left. -p, 1 - p, 3 - 2p, - \frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b + \sqrt{-a b}}] \right. \\
& \left. \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] \right) \Bigg/ \left(\left(b + \sqrt{-a b} \right) (2 - 2p) \right) \Bigg) - \\
& \frac{1}{4 (-1 + 2p)} \left((1 - 2p) \operatorname{AppellF1}[-2p, -p, -p, 1 - 2p, - \frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b + \sqrt{-a b}}, \right. \\
& \left. \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b + \sqrt{-a b}}] + 2p \operatorname{AppellF1}[1 - 2p, -p, -p, 2 - 2p, \right. \\
& \left. - \frac{(a+b) \operatorname{Sec}[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \operatorname{Sec}[c+d x]^2}{b + \sqrt{-a b}}] \operatorname{Sec}[c+d x]^2 \right) \\
& \left(\frac{-a + \sqrt{-a b} - (a+b) \operatorname{Tan}[c+d x]^2}{b + \sqrt{-a b}} \right)^{-p} \left(\frac{a + \sqrt{-a b} + (a+b) \operatorname{Tan}[c+d x]^2}{-b + \sqrt{-a b}} \right)^{-p} \\
& (\cos[c+d x]^4 (a + 2a \operatorname{Tan}[c+d x]^2 + (a+b) \operatorname{Tan}[c+d x]^4))^{-1+p} \\
& (\cos[c+d x]^4 (4a \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x] + 4 (a+b) \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]^3) - \\
& 4 \cos[c+d x]^3 \sin[c+d x] (a + 2a \operatorname{Tan}[c+d x]^2 + (a+b) \operatorname{Tan}[c+d x]^4)) \Bigg) \Bigg)
\end{aligned}$$

Problem 566: Result more than twice size of optimal antiderivative.

$$\int (a+b \sin[c+d x]^4)^p \operatorname{Tan}[c+d x] \, dx$$

Optimal (type 6, 141 leaves, 7 steps):

$$\left(\text{Hypergeometric2F1}\left[1, 1+p, 2+p, \frac{a+b \sin[c+d x]^4}{a+b}\right] (a+b \sin[c+d x]^4)^{1+p} \right) / \\ (4 (a+b) d (1+p)) + \frac{1}{2 d} \text{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, \sin[c+d x]^4, -\frac{b \sin[c+d x]^4}{a}\right] \\ \sin[c+d x]^2 (a+b \sin[c+d x]^4)^p \left(1 + \frac{b \sin[c+d x]^4}{a}\right)^{-p}$$

Result (type 6, 466 leaves):

$$- \left(\left(\left(-b + \sqrt{-a b}\right) \left(b + \sqrt{-a b}\right) (-1+2 p) \text{AppellF1}\left[-2 p, -p, -p, 1-2 p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}\right], \right. \right. \\ \left. \left. \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}\right) \cos[c+d x] \sin[c+d x] (a+b \sin[c+d x]^4)^p \right. \\ \left. \left(-a + \sqrt{-a b} - (a+b) \tan[c+d x]^2\right) \left(a + \sqrt{-a b} + (a+b) \tan[c+d x]^2\right)\right) / \\ \left(2 (a+b)^2 d p \left(b (-1+2 p) \text{AppellF1}\left[-2 p, -p, -p, 1-2 p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}\right], \right. \right. \\ \left. \left. \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}\right) \sin[2 (c+d x)] + 2 p \left(\left(b + \sqrt{-a b}\right) \text{AppellF1}\left[1-2 p, 1-p, \right. \right. \\ \left. \left.-p, 2-2 p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}\right] + \left(b - \sqrt{-a b}\right) \right. \\ \left. \left.\text{AppellF1}\left[1-2 p, -p, 1-p, 2-2 p, -\frac{(a+b) \sec[c+d x]^2}{-b + \sqrt{-a b}}, \frac{(a+b) \sec[c+d x]^2}{b + \sqrt{-a b}}\right]\right) \right. \\ \left. \tan[c+d x]\right) \left(a + 2 a \tan[c+d x]^2 + (a+b) \tan[c+d x]^4\right)\right)$$

Problem 568: Unable to integrate problem.

$$\int \cot[c+d x]^3 (a+b \sin[c+d x]^4)^p dx$$

Optimal (type 5, 127 leaves, 6 steps):

$$\frac{1}{4 a d (1+p)} \text{Hypergeometric2F1}\left[1, 1+p, 2+p, 1+\frac{b \sin[c+d x]^4}{a}\right] (a+b \sin[c+d x]^4)^{1+p} - \\ \frac{1}{2 d} \csc[c+d x]^2 \text{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \sin[c+d x]^4}{a}\right] \\ (a+b \sin[c+d x]^4)^p \left(1 + \frac{b \sin[c+d x]^4}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \cot[c+d x]^3 (a+b \sin[c+d x]^4)^p dx$$

Problem 574: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int (a + b \sin[c + d x]^n)^3 \tan[c + d x]^m dx$$

Optimal (type 5, 306 leaves, 10 steps):

$$\begin{aligned} & \frac{a^3 \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan[c+d x]^2\right] \tan[c+d x]^{1+m}}{d(1+m)} + \frac{1}{d(1+m+n)} \\ & 3 a^2 b (\cos[c+d x]^2)^{\frac{1-m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), \sin[c+d x]^2\right] \\ & \sin[c+d x]^n \tan[c+d x]^{1+m} + \frac{1}{d(1+m+2n)} \\ & 3 a b^2 (\cos[c+d x]^2)^{\frac{1-m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2}(1+m+2n), \frac{1}{2}(3+m+2n), \sin[c+d x]^2\right] \\ & \sin[c+d x]^{2n} \tan[c+d x]^{1+m} + \frac{1}{d(1+m+3n)} \\ & b^3 (\cos[c+d x]^2)^{\frac{1-m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2}(1+m+3n), \frac{1}{2}(3+m+3n), \sin[c+d x]^2\right] \\ & \sin[c+d x]^{3n} \tan[c+d x]^{1+m} \end{aligned}$$

Result (type 6, 13 001 leaves):

$$\begin{aligned} & \left(\left(a^3 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right. \right. \\ & \left. \left. \tan\left[\frac{1}{2}(c+d x)\right] \left(-\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{-1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^m \right) / \left((1+m) \left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2 \right) \right. \\ & \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \\ & 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \\ & \left. \left. m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \\ & \left. \tan\left[\frac{1}{2}(c+d x)\right]^2 \right) + \left(3 \times 2^n a^2 b (3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), \right. \right. \\ & \left. \left. m, 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right. \\ & \left. \tan\left[\frac{1}{2}(c+d x)\right] \left(-\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{-1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^m \left(\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^n \right) / \\ & \left((1+m+n) \left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2 \right) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, \right. \right. \right. \\ & \left. \left. \left. 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \end{aligned}$$

$$\begin{aligned}
& 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \right. \\
& -\tan \left[\frac{1}{2} (c+d x) \right]^2] -m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} \right. \\
& \left. \left. (5+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c+d x) \right]^2 \Big) + \\
& \left(3 \times 2^{2n} a b^2 (3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \right. \right. \\
& \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2] \tan \left[\frac{1}{2} (c+d x) \right] \\
& \left. \left. \left(-\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{-1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^{2n} \right) \right) / \\
& \left((1+m+2n) \left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), \right. \right. \right. \\
& m, 1+2n, \frac{1}{2} (3+m+2n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2] - \\
& 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), m, 2 (1+n), \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \right. \\
& -\tan \left[\frac{1}{2} (c+d x) \right]^2] -m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} \right. \\
& \left. \left. (5+m+2n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c+d x) \right]^2 \Big) + \\
& \left(2^{3n} b^3 (3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, 1+3n, \frac{1}{2} (3+m+3n), \right. \right. \\
& \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2] \tan \left[\frac{1}{2} (c+d x) \right] \\
& \left. \left. \left(-\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{-1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^{3n} \right) \right) / \\
& \left((1+m+3n) \left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \left((3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), \right. \right. \right. \\
& m, 1+3n, \frac{1}{2} (3+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2] - \\
& 2 \left((1+3n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), m, 2+3n, \frac{1}{2} (5+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \right. \\
& -\tan \left[\frac{1}{2} (c+d x) \right]^2] -m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), 1+m, 1+3n, \frac{1}{2} \right. \\
& \left. \left. (5+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c+d x) \right]^2 \Big) \Big) \\
& (a^3 \tan [c+d x]^m + 3 a^2 b \sin [c+d x]^n \tan [c+d x]^m + 3 a b^2 \sin [c+d x]^{2n} \\
& \tan [c+d x]^m + b^3 \\
& \sin [c+d x]^{3n})
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\operatorname{Tan}[c + d x]^m}{\operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right]} \right) / \\
& \left(d \left(- \left(\operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right. \right. \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \left(- \frac{\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^m}{-\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \right)^m \Bigg) / \\
& \quad \left((1+m) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right)^2 \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \right. \right. \right. \\
& \quad \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \Big] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \right. \right. \\
& \quad \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \Big] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \right. \right. \\
& \quad \left. \left. \left. \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \Bigg) + \\
& \quad \left(a^3 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \left(- \frac{\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^m}{-\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \right)^m \Bigg) / \left(2 (1+m) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right. \\
& \quad \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right] - \right. \\
& \quad 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right] - \right. \\
& \quad \left. \left. m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right] \right) \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) + \left(a^3 (3+m) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right. \\
& \quad \left(- \frac{1}{3+m} (1+m) \operatorname{AppellF1}\left[1 + \frac{1+m}{2}, m, 2, 1 + \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right] \right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] + \frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1 + \frac{1+m}{2}, 1+m, 1, \right. \\
& \quad \left. 1 + \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \Bigg) \\
& \quad \left. \left(- \frac{\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^m}{-\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \right)^m \right) / \left((1+m) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right. \\
& \quad \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\text{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] - \right. \\
& \quad m \text{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \Big) \\
& \quad \tan \left[\frac{1}{2} (c+d x) \right]^2 \Big) - \left(3 \times 2^n a^2 b (3+m+n) \text{AppellF1} \left[\frac{1}{2} (1+m+n), m, \right. \right. \\
& \quad 1+n, \frac{1}{2} (3+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2] \sec \left[\frac{1}{2} (c+d x) \right]^2 \\
& \quad \tan \left[\frac{1}{2} (c+d x) \right]^2 \left(-\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{-1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^n \Big) \Big) / \\
& \quad \left((1+m+n) \left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right)^2 \left((3+m+n) \text{AppellF1} \left[\frac{1}{2} (1+m+n), m, \right. \right. \right. \\
& \quad 1+n, \frac{1}{2} (3+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2] - \\
& \quad 2 \left((1+n) \text{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \\
& \quad -\tan \left[\frac{1}{2} (c+d x) \right]^2] - m \text{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} \right. \\
& \quad \left. \left. \left. (5+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) + \right. \\
& \quad \left(3 \times 2^{-1+n} a^2 b (3+m+n) \text{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \right. \right. \\
& \quad \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2] \sec \left[\frac{1}{2} (c+d x) \right]^2 \\
& \quad \left(-\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{-1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^n \Big) \Big) / \\
& \quad \left((1+m+n) \left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \left((3+m+n) \text{AppellF1} \left[\frac{1}{2} (1+m+n), m, \right. \right. \right. \\
& \quad 1+n, \frac{1}{2} (3+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2] - \\
& \quad 2 \left((1+n) \text{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \\
& \quad -\tan \left[\frac{1}{2} (c+d x) \right]^2] - m \text{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} \right. \\
& \quad \left. \left. \left. (5+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) + \right. \\
& \quad \left(3 \times 2^n a^2 b (3+m+n) \tan \left[\frac{1}{2} (c+d x) \right] \left(-\frac{1}{3+m+n} (1+n) (1+m+n) \right. \right. \\
& \quad \text{AppellF1} \left[1+\frac{1}{2} (1+m+n), m, 2+n, 1+\frac{1}{2} (3+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \\
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(c+d x)\right]^2 \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] + \frac{1}{3+m+n} \\
& m(1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), 1+m, 1+n, 1+\frac{1}{2}(3+m+n), \right. \\
& \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2 \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right]\right) \\
& \left(-\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{-1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^m \left(\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^n \Bigg) / \\
& \left((1+m+n) \left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2\right) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, \right. \right. \right. \\
& \left. \left. \left. 1+n, \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2} \right. \right. \right. \right. \\
& \left. \left. \left. \left. (5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \tan\left[\frac{1}{2}(c+d x)\right]^2\right) - \right. \\
& \left(3 \times 2^{2n} a b^2 (3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2 \sec\left[\frac{1}{2}(c+d x)\right]^2 \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2 \left(-\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{-1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^m \left(\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^{2n} \right) \right) / \right. \\
& \left((1+m+2n) \left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2\right)^2 \left((3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, \right. \right. \right. \\
& \left. \left. \left. 1+2n, \frac{1}{2}(3+m+2n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left((1+2n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), m, 2(1+n), \frac{1}{2}(5+m+2n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), 1+m, 1+2n, \frac{1}{2} \right. \right. \right. \right. \\
& \left. \left. \left. (5+m+2n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \tan\left[\frac{1}{2}(c+d x)\right]^2\right) + \right. \\
& \left(3 \times 2^{-1+2n} a b^2 (3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2 \sec\left[\frac{1}{2}(c+d x)\right]^2 \right. \right. \right. \\
& \left. \left. \left. \left(-\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{-1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^m \left(\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^{2n} \right) \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left((1 + m + 2 n) \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \left((3 + m + 2 n) \text{AppellF1} \left[\frac{1}{2} (1 + m + 2 n), \right. \right. \right. \\
& \quad \left. \left. \left. m, 1 + 2 n, \frac{1}{2} (3 + m + 2 n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left((1 + 2 n) \text{AppellF1} \left[\frac{1}{2} (3 + m + 2 n), m, 2 (1 + n), \frac{1}{2} (5 + m + 2 n), \tan \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - m \text{AppellF1} \left[\frac{1}{2} (3 + m + 2 n), 1 + m, 1 + 2 n, \frac{1}{2} \right. \right. \\
& \quad \left. \left. \left. (5 + m + 2 n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) + \\
& \left(3 \times 2^{2n} a b^2 (3 + m + 2 n) \tan \left[\frac{1}{2} (c + d x) \right] \left(-\frac{1}{3 + m + 2 n} (1 + 2 n) (1 + m + 2 n) \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[1 + \frac{1}{2} (1 + m + 2 n), m, 2 + 2 n, 1 + \frac{1}{2} (3 + m + 2 n), \tan \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] + \frac{1}{3 + m + 2 n} \right. \\
& \quad \left. m (1 + m + 2 n) \text{AppellF1} \left[1 + \frac{1}{2} (1 + m + 2 n), 1 + m, 1 + 2 n, 1 + \frac{1}{2} (3 + m + 2 n), \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \right) \\
& \quad \left(-\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^{2n} \Bigg) / \\
& \quad \left((1 + m + 2 n) \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \left((3 + m + 2 n) \text{AppellF1} \left[\frac{1}{2} (1 + m + 2 n), \right. \right. \right. \\
& \quad \left. \left. \left. m, 1 + 2 n, \frac{1}{2} (3 + m + 2 n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left((1 + 2 n) \text{AppellF1} \left[\frac{1}{2} (3 + m + 2 n), m, 2 (1 + n), \frac{1}{2} (5 + m + 2 n), \tan \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - m \text{AppellF1} \left[\frac{1}{2} (3 + m + 2 n), 1 + m, 1 + 2 n, \frac{1}{2} \right. \right. \\
& \quad \left. \left. \left. (5 + m + 2 n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) - \\
& \quad \left(2^{3n} b^3 (3 + m + 3 n) \text{AppellF1} \left[\frac{1}{2} (1 + m + 3 n), m, 1 + 3 n, \frac{1}{2} (3 + m + 3 n), \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right. \\
& \quad \left. \tan \left[\frac{1}{2} (c + d x) \right]^2 \left(-\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^{3n} \right) / \\
& \quad \left((1 + m + 3 n) \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \left((3 + m + 3 n) \text{AppellF1} \left[\frac{1}{2} (1 + m + 3 n), \right. \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \text{m, } 1+3n, \frac{1}{2}(3+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2] - \\
& 2 \left((1+3n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), m, 2+3n, \frac{1}{2}(5+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), 1+m, 1+3n, \frac{1}{2} \right. \right. \\
& \left. \left. (5+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+d x)\right]^2 \Big) + \\
& \left(2^{-1+3n} b^3 (3+m+3n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+3n), m, 1+3n, \frac{1}{2}(3+m+3n), \right. \right. \\
& \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2] \sec\left[\frac{1}{2}(c+d x)\right]^2 \\
& \left. \left. \left(-\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{-1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^m \left(\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^{3n} \right) \right) / \\
& \left((1+m+3n) \left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2 \right) \left((3+m+3n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+3n), \right. \right. \right. \\
& \left. \left. \left. m, 1+3n, \frac{1}{2}(3+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \\
& 2 \left((1+3n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), m, 2+3n, \frac{1}{2}(5+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), 1+m, 1+3n, \frac{1}{2} \right. \right. \\
& \left. \left. (5+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+d x)\right]^2 \Big) + \\
& \left(2^{3n} b^3 (3+m+3n) \tan\left[\frac{1}{2}(c+d x)\right] \left(-\frac{1}{3+m+3n} (1+3n) (1+m+3n) \right. \right. \\
& \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+3n), m, 2+3n, 1+\frac{1}{2}(3+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \\
& \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2] \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] + \frac{1}{3+m+3n} \right. \\
& m (1+m+3n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+3n), 1+m, 1+3n, 1+\frac{1}{2}(3+m+3n), \right. \\
& \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2] \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] \Big) \\
& \left. \left(-\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{-1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^m \left(\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^{3n} \right) / \\
& \left((1+m+3n) \left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2 \right) \left((3+m+3n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+3n), \right. \right. \right. \\
& \left. \left. \left. m, 1+3n, \frac{1}{2}(3+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \\
& 2 \left((1+3n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), m, 2+3n, \frac{1}{2}(5+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+3n), 1+m, 1+3n, \frac{1}{2} \right. \right. \\
& \left. \left. (5+m+3n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+d x)\right]^2 \Big)
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2} (c+d x)^2\right] - m \text{AppellF1}\left[\frac{1}{2} (3+m+3 n), 1+m, 1+3 n, \frac{1}{2} \right. \\
& \quad \left. (5+m+3 n), \tan\left[\frac{1}{2} (c+d x)^2\right], -\tan\left[\frac{1}{2} (c+d x)^2\right]\right) \tan\left[\frac{1}{2} (c+d x)^2\right] + \\
& \left(a^3 m (3+m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2} (c+d x)^2\right], -\tan\left[\frac{1}{2} (c+d x)^2\right]\right] \right. \\
& \quad \left. \tan\left[\frac{1}{2} (c+d x)\right] \left(-\frac{\tan\left[\frac{1}{2} (c+d x)\right]}{-1+\tan\left[\frac{1}{2} (c+d x)\right]^2} \right)^{-1+m} \right. \\
& \quad \left. \left(\frac{\sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right]^2}{(-1+\tan\left[\frac{1}{2} (c+d x)\right]^2)^2} - \frac{\sec\left[\frac{1}{2} (c+d x)\right]^2}{2 (-1+\tan\left[\frac{1}{2} (c+d x)\right]^2)} \right) \right) / \\
& \quad \left((1+m) \left(1+\tan\left[\frac{1}{2} (c+d x)\right]^2 \right) \right. \\
& \quad \left. \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left(\text{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - \right. \right. \\
& \quad \left. \left. m \text{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \right) \tan\left[\frac{1}{2} (c+d x)\right]^2 \right) + \\
& \left(3 \times 2^n a^2 b m (3+m+n) \text{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \tan\left[\frac{1}{2} (c+d x)\right] \right. \\
& \quad \left. \left(-\frac{\tan\left[\frac{1}{2} (c+d x)\right]}{-1+\tan\left[\frac{1}{2} (c+d x)\right]^2} \right)^{-1+m} \left(\frac{\tan\left[\frac{1}{2} (c+d x)\right]}{1+\tan\left[\frac{1}{2} (c+d x)\right]^2} \right)^n \right. \\
& \quad \left. \left(\frac{\sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right]^2}{(-1+\tan\left[\frac{1}{2} (c+d x)\right]^2)^2} - \frac{\sec\left[\frac{1}{2} (c+d x)\right]^2}{2 (-1+\tan\left[\frac{1}{2} (c+d x)\right]^2)} \right) \right) / \\
& \quad \left((1+m+n) \left(1+\tan\left[\frac{1}{2} (c+d x)\right]^2 \right) \left((3+m+n) \text{AppellF1}\left[\frac{1}{2} (1+m+n), m, \right. \right. \right. \\
& \quad \left. \left. \left. 1+n, \frac{1}{2} (3+m+n), \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \text{AppellF1}\left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \tan\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - m \text{AppellF1}\left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} \right. \right. \\
& \quad \left. \left. (5+m+n), \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \right) \tan\left[\frac{1}{2} (c+d x)\right]^2 \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(3 \times 2^{2n} ab^2 m (3 + m + 2n) \text{AppellF1}\left[\frac{1}{2} (1 + m + 2n), m, 1 + 2n, \frac{1}{2} (3 + m + 2n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right] \tan\left[\frac{1}{2} (c + dx)\right] \right. \\
& \quad \left. \left(-\frac{\tan\left[\frac{1}{2} (c + dx)\right]}{-1 + \tan\left[\frac{1}{2} (c + dx)\right]^2} \right)^{-1+m} \left(\frac{\tan\left[\frac{1}{2} (c + dx)\right]}{1 + \tan\left[\frac{1}{2} (c + dx)\right]^2} \right)^{2n} \right. \\
& \quad \left. \left(\frac{\sec\left[\frac{1}{2} (c + dx)\right]^2 \tan\left[\frac{1}{2} (c + dx)\right]^2}{(-1 + \tan\left[\frac{1}{2} (c + dx)\right]^2)^2} - \frac{\sec\left[\frac{1}{2} (c + dx)\right]^2}{2 (-1 + \tan\left[\frac{1}{2} (c + dx)\right]^2)} \right) \right) / \\
& \quad \left((1 + m + 2n) \left(1 + \tan\left[\frac{1}{2} (c + dx)\right]^2 \right) \left((3 + m + 2n) \text{AppellF1}\left[\frac{1}{2} (1 + m + 2n), \right. \right. \right. \\
& \quad \left. \left. \left. m, 1 + 2n, \frac{1}{2} (3 + m + 2n), \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1 + 2n) \text{AppellF1}\left[\frac{1}{2} (3 + m + 2n), m, 2 (1 + n), \frac{1}{2} (5 + m + 2n), \tan\left[\frac{1}{2} (c + dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (c + dx)\right]^2 \right] - m \text{AppellF1}\left[\frac{1}{2} (3 + m + 2n), 1 + m, 1 + 2n, \frac{1}{2} \right. \right. \\
& \quad \left. \left. \left. (5 + m + 2n), \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2 \right] \right) \tan\left[\frac{1}{2} (c + dx)\right]^2 \right) \right) + \\
& \quad \left(2^{3n} b^3 m (3 + m + 3n) \text{AppellF1}\left[\frac{1}{2} (1 + m + 3n), m, 1 + 3n, \frac{1}{2} (3 + m + 3n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right] \tan\left[\frac{1}{2} (c + dx)\right] \right. \\
& \quad \left. \left(-\frac{\tan\left[\frac{1}{2} (c + dx)\right]}{-1 + \tan\left[\frac{1}{2} (c + dx)\right]^2} \right)^{-1+m} \left(\frac{\tan\left[\frac{1}{2} (c + dx)\right]}{1 + \tan\left[\frac{1}{2} (c + dx)\right]^2} \right)^{3n} \right. \\
& \quad \left. \left(\frac{\sec\left[\frac{1}{2} (c + dx)\right]^2 \tan\left[\frac{1}{2} (c + dx)\right]^2}{(-1 + \tan\left[\frac{1}{2} (c + dx)\right]^2)^2} - \frac{\sec\left[\frac{1}{2} (c + dx)\right]^2}{2 (-1 + \tan\left[\frac{1}{2} (c + dx)\right]^2)} \right) \right) / \\
& \quad \left((1 + m + 3n) \left(1 + \tan\left[\frac{1}{2} (c + dx)\right]^2 \right) \left((3 + m + 3n) \text{AppellF1}\left[\frac{1}{2} (1 + m + 3n), \right. \right. \right. \\
& \quad \left. \left. \left. m, 1 + 3n, \frac{1}{2} (3 + m + 3n), \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1 + 3n) \text{AppellF1}\left[\frac{1}{2} (3 + m + 3n), m, 2 + 3n, \frac{1}{2} (5 + m + 3n), \tan\left[\frac{1}{2} (c + dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (c + dx)\right]^2 \right] - m \text{AppellF1}\left[\frac{1}{2} (3 + m + 3n), 1 + m, 1 + 3n, \frac{1}{2} \right. \right. \\
& \quad \left. \left. \left. (5 + m + 3n), \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2 \right] \right) \tan\left[\frac{1}{2} (c + dx)\right]^2 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(3 \times 2^n a^2 b n (3 + m + n) \text{AppellF1} \left[\frac{1}{2} (1 + m + n), m, 1 + n, \frac{1}{2} (3 + m + n), \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \tan \left[\frac{1}{2} (c + d x) \right] \right. \\
& \quad \left. \left(-\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^{-1+n} \right. \\
& \quad \left. \left. \left(-\frac{\sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right]^2}{(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2)^2} + \frac{\sec \left[\frac{1}{2} (c + d x) \right]^2}{2 (1 + \tan \left[\frac{1}{2} (c + d x) \right]^2)} \right) \right) / \right. \\
& \quad \left. \left((1 + m + n) \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \left((3 + m + n) \text{AppellF1} \left[\frac{1}{2} (1 + m + n), m, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 1 + n, \frac{1}{2} (3 + m + n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. \left. 2 \left((1 + n) \text{AppellF1} \left[\frac{1}{2} (3 + m + n), m, 2 + n, \frac{1}{2} (5 + m + n), \tan \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - m \text{AppellF1} \left[\frac{1}{2} (3 + m + n), 1 + m, 1 + n, \frac{1}{2} \right. \right. \\
& \quad \left. \left. \left. \left. (5 + m + n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) + \right. \\
& \quad \left. \left(3 \times 2^{1+2n} a b^2 n (3 + m + 2n) \text{AppellF1} \left[\frac{1}{2} (1 + m + 2n), m, 1 + 2n, \frac{1}{2} (3 + m + 2n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \tan \left[\frac{1}{2} (c + d x) \right] \right. \\
& \quad \left. \left. \left(-\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^{-1+2n} \right. \right. \\
& \quad \left. \left. \left(-\frac{\sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right]^2}{(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2)^2} + \frac{\sec \left[\frac{1}{2} (c + d x) \right]^2}{2 (1 + \tan \left[\frac{1}{2} (c + d x) \right]^2)} \right) \right) / \right. \\
& \quad \left. \left. \left((1 + m + 2n) \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \left((3 + m + 2n) \text{AppellF1} \left[\frac{1}{2} (1 + m + 2n), \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. m, 1 + 2n, \frac{1}{2} (3 + m + 2n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. \left. 2 \left((1 + 2n) \text{AppellF1} \left[\frac{1}{2} (3 + m + 2n), m, 2 (1 + n), \frac{1}{2} (5 + m + 2n), \tan \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - m \text{AppellF1} \left[\frac{1}{2} (3 + m + 2n), 1 + m, 1 + 2n, \frac{1}{2} \right. \right. \\
& \quad \left. \left. \left. \left. (5 + m + 2n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) + \right)
\end{aligned}$$

$$\begin{aligned}
& \left(3 \times 2^{3n} b^3 n (3 + m + 3n) \operatorname{AppellF1} \left[\frac{1}{2} (1 + m + 3n), m, 1 + 3n, \frac{1}{2} (3 + m + 3n), \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \tan \left[\frac{1}{2} (c + d x) \right] \right. \\
& \quad \left. \left(-\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^m \left(\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^{-1+3n} \right. \\
& \quad \left. \left. \left(-\frac{\sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right]^2}{\left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2} + \frac{\sec \left[\frac{1}{2} (c + d x) \right]^2}{2 \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)} \right) \right) / \right. \\
& \quad \left. \left((1 + m + 3n) \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \left((3 + m + 3n) \operatorname{AppellF1} \left[\frac{1}{2} (1 + m + 3n), \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. m, 1 + 3n, \frac{1}{2} (3 + m + 3n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1 + 3n) \operatorname{AppellF1} \left[\frac{1}{2} (3 + m + 3n), m, 2 + 3n, \frac{1}{2} (5 + m + 3n), \tan \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{1}{2} (3 + m + 3n), 1 + m, 1 + 3n, \frac{1}{2} \right. \right. \\
& \quad \left. \left. \left. (5 + m + 3n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) - \\
& \quad \left(a^3 (3 + m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right. \\
& \quad \left. \tan \left[\frac{1}{2} (c + d x) \right] \left(-\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^m \right. \\
& \quad \left. \left(-2 \left(\operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \right. \\
& \quad \left. \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] + (3 + m) \left(-\frac{1}{3 + m} (1 + m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. m, 2, 1 + \frac{3+m}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{2} (c + d x) \right] + \frac{1}{3 + m} m (1 + m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1 + m, 1, 1 + \frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \right) - \right. \\
& \quad \left. 2 \tan \left[\frac{1}{2} (c + d x) \right]^2 \left(-\frac{1}{5 + m} 2 (3 + m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, m, 3, 1 + \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] + \right. \right. \\
& \quad \left. \left. \left. \frac{1}{5 + m} m (3 + m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1 + m, 2, 1 + \frac{5+m}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\begin{aligned} & \frac{1}{2} \operatorname{Tan}^2[c+d x], -\operatorname{Tan}^2[\frac{1}{2}(c+d x)] \operatorname{Sec}^2[\frac{1}{2}(c+d x)]^2 \operatorname{Tan}[\frac{1}{2}(c+d x)] + \\ & \frac{1}{5+m+n} m (3+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+n), 1+m, 2+n, 1+\frac{1}{2}(5+m+n), \right. \\ & \left. \operatorname{Tan}^2[\frac{1}{2}(c+d x)], -\operatorname{Tan}^2[\frac{1}{2}(c+d x)] \operatorname{Sec}^2[\frac{1}{2}(c+d x)]^2 \operatorname{Tan}[\frac{1}{2}(c+d x)] \right] - \\ & m \left(-\frac{1}{5+m+n} (1+n) (3+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+n), 1+m, 2+n, \right. \right. \\ & \left. \left. 1+\frac{1}{2}(5+m+n), \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2, -\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right] \right. \\ & \left. \operatorname{Sec}^2[\frac{1}{2}(c+d x)]^2 \operatorname{Tan}[\frac{1}{2}(c+d x)] + \frac{1}{5+m+n} (1+m) (3+m+n) \right. \\ & \left. \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+n), 2+m, 1+n, 1+\frac{1}{2}(5+m+n), \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2, \right. \right. \\ & \left. \left. -\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right] \operatorname{Sec}^2[\frac{1}{2}(c+d x)]^2 \operatorname{Tan}[\frac{1}{2}(c+d x)] \right) \right) \right) / \\ & \left((1+m+n) \left(1 + \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \right. \right. \right. \\ & \left. \left. \left. \frac{1}{2}(3+m+n), \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2, -\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right] - \right. \\ & \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2, \right. \right. \right. \\ & \left. \left. \left. -\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right] - m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \right. \right. \right. \\ & \left. \left. \left. \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2, -\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right] \right) \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right) - \\ & \left(3 \times 2^{2n} a b^2 (3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, 1+2n, \frac{1}{2}(3+m+2n), \right. \right. \\ & \left. \left. \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2, -\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right] \operatorname{Tan}[\frac{1}{2}(c+d x)] \right. \\ & \left. \left(-\frac{\operatorname{Tan}[\frac{1}{2}(c+d x)]}{-\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2} \right)^m \left(\frac{\operatorname{Tan}[\frac{1}{2}(c+d x)]}{1+\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2} \right)^{2n} \right. \\ & \left. \left(-2 \left((1+2n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), m, 2(1+n), \frac{1}{2}(5+m+2n), \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2, \right. \right. \right. \right. \\ & \left. \left. \left. \left. -\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right] - m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), 1+m, 1+2n, \frac{1}{2}(5+m+2n), \right. \right. \right. \\ & \left. \left. \left. \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2, -\operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2 \right] \right) \operatorname{Sec}^2[\frac{1}{2}(c+d x)]^2 \operatorname{Tan}[\frac{1}{2}(c+d x)] + \right. \\ & \left. (3+m+2n) \left(-\frac{1}{3+m+2n} (1+2n) (1+m+2n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+2n), \right. \right. \right. \\ & \left. \left. \left. m, 2+2n, 1+\frac{1}{2}(3+m+2n), \operatorname{Tan}^2[\frac{1}{2}(c+d x)]^2, \right. \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(c+dx)\right]^2] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3+m+2n} \\
& m(1+m+2n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+2n), 1+m, 1+2n, 1+\frac{1}{2}(3+m+2n), \right. \\
& \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right] - \\
& 2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left((1+2n) \left(-\frac{1}{5+m+2n} 2(1+n)(3+m+2n) \operatorname{AppellF1}\left[\right. \right. \right. \\
& \left. \left. \left. 1+\frac{1}{2}(3+m+2n), m, 1+2(1+n), 1+\frac{1}{2}(5+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{5+m+2n} \right. \right. \right. \\
& \left. \left. \left. m(3+m+2n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+2n), 1+m, 2(1+n), 1+\frac{1}{2}(5+m+2n), \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right] - \right. \right. \\
& \left. \left. m \left(-\frac{1}{5+m+2n}(1+2n)(3+m+2n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+2n), 1+m, \right. \right. \right. \\
& \left. \left. \left. 2+2n, 1+\frac{1}{2}(5+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2] \right. \right. \right. \\
& \left. \left. \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{5+m+2n}(1+m)(3+m+2n) \operatorname{AppellF1}\left[\right. \right. \right. \\
& \left. \left. \left. 1+\frac{1}{2}(3+m+2n), 2+m, 1+2n, 1+\frac{1}{2}(5+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) \right) \Bigg) / \\
& \left((1+m+2n) \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \left((3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+2n), m, \right. \right. \right. \\
& \left. \left. \left. 1+2n, \frac{1}{2}(3+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left((1+2n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), m, 2(1+n), \frac{1}{2}(5+m+2n), \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2] - m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+2n), 1+m, 1+2n, \frac{1}{2}(5+m+2n), \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) - \right. \\
& \left. \left(2^{3n} b^3 (3+m+3n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+3n), m, 1+3n, \frac{1}{2}(3+m+3n), \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2] \tan\left[\frac{1}{2}(c+dx)\right] \right. \right. \right. \\
& \left. \left. \left. \left(-\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{-1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^m \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3n} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-2 \left((1+3n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), m, 2+3n, \frac{1}{2} (5+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] -m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), 1+m, 1+3n, \frac{1}{2} (5+m+3n), \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \right) \sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right] + \right. \\
& \quad (3+m+3n) \left(-\frac{1}{3+m+3n} (1+3n) (1+m+3n) \operatorname{AppellF1} \left[1+\frac{1}{2} (1+m+3n), \right. \right. \\
& \quad \left. \left. m, 2+3n, 1+\frac{1}{2} (3+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right] + \frac{1}{3+m+3n} \right. \\
& \quad \left. m (1+m+3n) \operatorname{AppellF1} \left[1+\frac{1}{2} (1+m+3n), 1+m, 1+3n, 1+\frac{1}{2} (3+m+3n), \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right] \right) - \\
& \quad 2 \tan \left[\frac{1}{2} (c+d x) \right]^2 \left((1+3n) \left(-\frac{1}{5+m+3n} (2+3n) (3+m+3n) \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1}{2} (3+m+3n), m, 3+3n, 1+\frac{1}{2} (5+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right] + \frac{1}{5+m+3n} \right. \\
& \quad \left. m (3+m+3n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+3n), 1+m, 2+3n, 1+\frac{1}{2} (5+m+3n), \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right] \right) - \\
& \quad \left. m \left(-\frac{1}{5+m+3n} (1+3n) (3+m+3n) \operatorname{AppellF1} \left[1+\frac{1}{2} (3+m+3n), 1+m, \right. \right. \right. \\
& \quad \left. \left. \left. 2+3n, 1+\frac{1}{2} (5+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right] + \frac{1}{5+m+3n} (1+m) (3+m+3n) \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1}{2} (3+m+3n), 2+m, 1+3n, 1+\frac{1}{2} (5+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right] \right) \right) \right) \right) \right) \right) / \\
& \quad \left((1+m+3n) \left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \left((3+m+3n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+3n), m, \right. \right. \right. \\
& \quad \left. \left. \left. 1+3n, \frac{1}{2} (3+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+3n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), m, 2+3n, \frac{1}{2} (5+m+3n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] -m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+3n), 1+m, 1+3n, \frac{1}{2} (5+m+3n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \right) \right) \right) \right) \right) \right)
\end{aligned}$$

$$\left. \left(\frac{1}{2} \left(c + d x \right)^2, -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right) \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2 \right) \right)$$

Problem 575: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b \sin[c + d x]^n)^2 \tan[c + d x]^m dx$$

Optimal (type 5, 215 leaves, 8 steps):

$$\begin{aligned} & \frac{a^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan[c+d x]^2\right] \tan[c+d x]^{1+m}}{d (1+m)} + \frac{1}{d (1+m+n)} \\ & 2 a b (\cos[c+d x]^2)^{\frac{1+m}{2}} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2} (1+m+n), \frac{1}{2} (3+m+n), \sin[c+d x]^2\right] \\ & \sin[c+d x]^n \tan[c+d x]^{1+m} + \frac{1}{d (1+m+2n)} \\ & b^2 (\cos[c+d x]^2)^{\frac{1+m}{2}} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2} (1+m+2n), \frac{1}{2} (3+m+2n), \sin[c+d x]^2\right] \\ & \sin[c+d x]^{2n} \tan[c+d x]^{1+m} \end{aligned}$$

Result (type 6, 8343 leaves):

$$\begin{aligned} & \left(2^{1+m} \tan\left[\frac{1}{2} (c+d x)\right] \left(-\frac{\tan\left[\frac{1}{2} (c+d x)\right]}{-1 + \tan\left[\frac{1}{2} (c+d x)\right]^2} \right)^m \right. \\ & \left. \left(\left(a^2 (3+m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \right) / \right. \right. \\ & \left. \left. \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \right) - \right. \right. \\ & \left. \left. 2 \left(\text{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - \right. \right. \\ & \left. \left. m \text{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \right) \right. \\ & \left. \tan\left[\frac{1}{2} (c+d x)\right]^2 \right) + \left(2^{1+n} a b (3+m+n) \text{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \right. \right. \\ & \left. \left. \frac{1}{2} (3+m+n), \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2 \right] \left(\frac{\tan\left[\frac{1}{2} (c+d x)\right]}{1 + \tan\left[\frac{1}{2} (c+d x)\right]^2} \right)^n \right) / \\ & \left((1+m+n) \left((3+m+n) \text{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \tan\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \\ & \left. \left. \left. -\tan\left[\frac{1}{2} (c+d x)\right]^2 \right] - 2 \left((1+n) \text{AppellF1}\left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \right. \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. \left(\begin{aligned}
& \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2] - m \text{AppellF1} \left[\frac{1}{2} (3 + m + n), 1 + m, 1 + \right. \\
& \left. n, \frac{1}{2} (5 + m + n), \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2] \right) \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) + \\
& \left(4^n b^2 (3 + m + 2 n) \text{AppellF1} \left[\frac{1}{2} (1 + m + 2 n), m, 1 + 2 n, \frac{1}{2} (3 + m + 2 n), \right. \right. \\
& \left. \left. \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \left(\frac{\text{Tan} \left[\frac{1}{2} (c + d x) \right]}{1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \right)^{2 n} \right) / \\
& \left((1 + m + 2 n) \left((3 + m + 2 n) \text{AppellF1} \left[\frac{1}{2} (1 + m + 2 n), m, 1 + 2 n, \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} (3 + m + 2 n), \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \\
& \left. 2 \left((1 + 2 n) \text{AppellF1} \left[\frac{1}{2} (3 + m + 2 n), m, 2 (1 + n), \frac{1}{2} (5 + m + 2 n), \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \\
& \left. \left. \left. -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] - m \text{AppellF1} \left[\frac{1}{2} (3 + m + 2 n), 1 + m, 1 + 2 n, \frac{1}{2} \right. \right. \\
& \left. \left. \left. (5 + m + 2 n), \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \\
& (a^2 \text{Tan} [c + d x]^m + 2 a b \sin [c + d x]^n \text{Tan} [c + d x]^m + b^2 \sin [c + d x]^{2 n} \\
& \left. \left. \text{Tan} [c + d x]^m \right) \right) / \\
& \left(d \left(1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \left(- \frac{1}{\left(1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2} \right. \right. \\
& \left. \left. 2^{1+m} \sec \left[\frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \left(- \frac{\text{Tan} \left[\frac{1}{2} (c + d x) \right]}{-1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \right)^m \right. \right. \\
& \left. \left. \left(\left(a^2 (3 + m) \text{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \right. \right) / \right. \\
& \left. \left. \left((1 + m) \left((3 + m) \text{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] - 2 \left(\text{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] - m \text{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) + \right. \\
& \left. \left. \left(2^{1+n} a b (3 + m + n) \text{AppellF1} \left[\frac{1}{2} (1 + m + n), m, 1 + n, \frac{1}{2} (3 + m + n), \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{2} (c+d x)^2\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2} \right)^n \Bigg) \Bigg/ \left((1+m+n) \left((3+m+n) \operatorname{AppellF1}\left[\right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - \right. \\
& \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - m \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \right. \right. \\
& \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right) + \\
& \left(4^n b^2 (3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2} \right)^{2n} \right) \Bigg/ \\
& \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - 2 \left((1+2n) \operatorname{AppellF1}\left[\frac{1}{2} (3+m+2n), \right. \right. \right. \\
& \left. \left. \left. m, 2 (1+n), \frac{1}{2} (5+m+2n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - \right. \\
& \left. \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right) \Bigg) + \\
& \frac{1}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2} 2^m \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \left(-\frac{\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2} \right)^m \\
& \left(\left(a^2 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right) \right. \\
& \left. \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - \right. \right. \right. \\
& \left. \left. \left. 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - \right. \right. \right. \\
& \left. \left. \left. m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right) \right) \\
& \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right) + \left(2^{1+n} a b (3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} (3 + m + n), \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2] \left(\frac{\tan\left[\frac{1}{2} (c + d x)\right]}{1 + \tan\left[\frac{1}{2} (c + d x)\right]^2} \right)^n \Bigg) / \\
& \left((1 + m + n) \left((3 + m + n) \text{AppellF1}\left[\frac{1}{2} (1 + m + n), m, 1 + n, \frac{1}{2} (3 + m + n), \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] - \right. \right. \\
& \quad 2 \left((1 + n) \text{AppellF1}\left[\frac{1}{2} (3 + m + n), m, 2 + n, \frac{1}{2} (5 + m + n), \tan\left[\frac{1}{2} (c + d x)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] - m \text{AppellF1}\left[\frac{1}{2} (3 + m + n), 1 + m, 1 + n, \frac{1}{2} (5 + m + n), \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] \right) \tan\left[\frac{1}{2} (c + d x)\right]^2 \Bigg) + \\
& \left(4^n b^2 (3 + m + 2 n) \text{AppellF1}\left[\frac{1}{2} (1 + m + 2 n), m, 1 + 2 n, \frac{1}{2} (3 + m + 2 n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] \left(\frac{\tan\left[\frac{1}{2} (c + d x)\right]}{1 + \tan\left[\frac{1}{2} (c + d x)\right]^2} \right)^{2 n} \right) / \\
& \left((1 + m + 2 n) \left((3 + m + 2 n) \text{AppellF1}\left[\frac{1}{2} (1 + m + 2 n), m, 1 + 2 n, \frac{1}{2} (3 + m + 2 n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] - 2 \left((1 + 2 n) \text{AppellF1}\left[\frac{1}{2} (3 + m + 2 n), \right. \right. \right. \\
& \quad \left. \left. \left. m, 2 (1 + n), \frac{1}{2} (5 + m + 2 n), \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. m \text{AppellF1}\left[\frac{1}{2} (3 + m + 2 n), 1 + m, 1 + 2 n, \frac{1}{2} (5 + m + 2 n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] \right) \tan\left[\frac{1}{2} (c + d x)\right]^2 \right) \Bigg) + \\
& \frac{1}{1 + \tan\left[\frac{1}{2} (c + d x)\right]^2} 2^{1+m} m \tan\left[\frac{1}{2} (c + d x)\right] \left(-\frac{\tan\left[\frac{1}{2} (c + d x)\right]}{-1 + \tan\left[\frac{1}{2} (c + d x)\right]^2} \right)^{-1+m} \\
& \left(\frac{\sec\left[\frac{1}{2} (c + d x)\right]^2 \tan\left[\frac{1}{2} (c + d x)\right]^2}{(-1 + \tan\left[\frac{1}{2} (c + d x)\right]^2)^2} - \frac{\sec\left[\frac{1}{2} (c + d x)\right]^2}{2 (-1 + \tan\left[\frac{1}{2} (c + d x)\right]^2)} \right) \\
& \left(a^2 (3 + m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] \right) / \\
& \left((1 + m) \left((3 + m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left(\text{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c + d x)\right]^2, -\tan\left[\frac{1}{2} (c + d x)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. m \text{AppellF1}\left[\frac{3+m}{2}, 1 + m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c + d x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2} (c + d x)\right]^2 \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& - \operatorname{Tan}\left[\frac{1}{2} (c + d x)^2\right] \operatorname{Tan}\left[\frac{1}{2} (c + d x)^2\right] \Big) + \\
& \left(2^{1+n} a b (3 + m + n) \operatorname{AppellF1}\left[\frac{1}{2} (1 + m + n), m, 1 + n, \frac{1}{2} (3 + m + n), \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c + d x)^2\right], -\operatorname{Tan}\left[\frac{1}{2} (c + d x)^2\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2} \right)^n \right) \right) / \\
& \left((1 + m + n) \left((3 + m + n) \operatorname{AppellF1}\left[\frac{1}{2} (1 + m + n), m, 1 + n, \frac{1}{2} (3 + m + n), \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c + d x)^2\right], -\operatorname{Tan}\left[\frac{1}{2} (c + d x)^2\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 \left((1 + n) \operatorname{AppellF1}\left[\frac{1}{2} (3 + m + n), m, 2 + n, \frac{1}{2} (5 + m + n), \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right] - m \operatorname{AppellF1}\left[\frac{1}{2} (3 + m + n), 1 + m, 1 + n, \frac{1}{2} (5 + m + n), \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2} (c + d x)^2\right] \right) + \\
& \left(4^n b^2 (3 + m + 2 n) \operatorname{AppellF1}\left[\frac{1}{2} (1 + m + 2 n), m, 1 + 2 n, \frac{1}{2} (3 + m + 2 n), \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2} \right)^{2n} \right) / \\
& \left((1 + m + 2 n) \left((3 + m + 2 n) \operatorname{AppellF1}\left[\frac{1}{2} (1 + m + 2 n), m, 1 + 2 n, \frac{1}{2} (3 + m + 2 n), \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right] - 2 \left((1 + 2 n) \operatorname{AppellF1}\left[\frac{1}{2} (3 + m + 2 n), \right. \right. \right. \\
& \quad \left. \left. \left. m, 2 (1 + n), \frac{1}{2} (5 + m + 2 n), \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1}\left[\frac{1}{2} (3 + m + 2 n), 1 + m, 1 + 2 n, \frac{1}{2} (5 + m + 2 n), \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right] \right) \right) + \\
& \left. \left. \left. \frac{1}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2} 2^{1+m} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{-1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2} \right)^m \right. \right. \right. \\
& \quad \left(\left(a^2 (3 + m) \left(-\frac{1}{3 + m} (1 + m) \operatorname{AppellF1}\left[1 + \frac{1 + m}{2}, m, 2, 1 + \frac{3 + m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \frac{1}{3 + m} \right. \right. \right. \\
& \quad \left. \left. \left. m (1 + m) \operatorname{AppellF1}\left[1 + \frac{1 + m}{2}, 1 + m, 1, 1 + \frac{3 + m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, \right. \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2] \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right)\right)\Bigg) \\
& \left(\left(1+m\right) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right]-\right.\right. \\
& 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right]-\right. \\
& \left.m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2,\right.\right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\Bigg)+ \\
& \left(2^{1+n} a b (3+m+n) \left(-\frac{1}{3+m+n} (1+n) (1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2} (1+m+n), m, 2+n,\right.\right.\right. \\
& \left.\left.\left.1+\frac{1}{2} (3+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2\right.\right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]+\frac{1}{3+m+n} m (1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2} (1+m+n),\right.\right.\right. \\
& \left.\left.\left.1+m, 1+n, 1+\frac{1}{2} (3+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right]\right.\right. \\
& \left.\left.\operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right)\left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}\right)^n\right)\Bigg) \\
& \left(\left(1+m+n\right) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n),\right.\right.\right. \\
& \left.\left.\left.\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right]-\right.\right. \\
& 2 \left(\left(1+n\right) \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2,\right.\right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right]-m \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n),\right.\right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)+ \\
& \left(4^n b^2 (3+m+2 n) \left(-\frac{1}{3+m+2 n} (1+2 n) (1+m+2 n) \operatorname{AppellF1}\left[1+\frac{1}{2} (1+m+2 n),\right.\right.\right. \\
& \left.\left.\left.m, 2+2 n, 1+\frac{1}{2} (3+m+2 n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2,\right.\right.\right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]+\right. \\
& \left.\frac{1}{3+m+2 n} m (1+m+2 n) \operatorname{AppellF1}\left[1+\frac{1}{2} (1+m+2 n), 1+m, 1+2 n,\right.\right. \\
& \left.\left.1+\frac{1}{2} (3+m+2 n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right)\left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}\right)^{2 n}\right)\Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] - 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), \right. \right. \right. \\
& \quad \left. \left. \left. m, 2 (1+n), \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] - \right. \\
& \quad \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) + \\
& \left(2^{1+n} a b n (3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \left(\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^{-1+n} \right. \\
& \quad \left. \left(-\frac{\sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right]^2}{\left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right)^2} + \frac{\sec \left[\frac{1}{2} (c+d x) \right]^2}{2 \left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right)} \right) \right) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left((1+n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} (c+d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) + \\
& \left(2^{1+2n} b^2 n (3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \left(\frac{\tan \left[\frac{1}{2} (c+d x) \right]}{1+\tan \left[\frac{1}{2} (c+d x) \right]^2} \right)^{-1+2n} \right. \\
& \quad \left. \left(-\frac{\sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right]^2}{\left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right)^2} + \frac{\sec \left[\frac{1}{2} (c+d x) \right]^2}{2 \left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right)} \right) \right) / \\
& \left((1+m+2n) \left((3+m+2n) \operatorname{AppellF1} \left[\frac{1}{2} (1+m+2n), m, 1+2n, \frac{1}{2} (3+m+2n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] - 2 \left((1+2n) \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), \right. \right. \right. \\
& \quad \left. \left. \left. m, 2 (1+n), \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] - \right. \\
& \quad \left. \left. \left. m \operatorname{AppellF1} \left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \tan \left[\frac{1}{2} (c+d x) \right]^2, -\tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& m \operatorname{AppellF1}\left[\frac{1}{2} (3+m+2n), 1+m, 1+2n, \frac{1}{2} (5+m+2n), \right. \\
& \quad \left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \tan\left[\frac{1}{2} (c+d x)\right]^2 \Big) - \\
& \left(a^2 (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \right. \\
& \quad \left(-2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right]\right) \right. \\
& \quad \left. \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] + (3+m) \left(-\frac{1}{3+m} (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. m, 2, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2 \right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (c+d x)\right] + \frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1+m, 1, 1+\frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2 \right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] \right) - \right. \\
& \quad \left. 2 \tan\left[\frac{1}{2} (c+d x)\right]^2 \left(-\frac{1}{5+m} 2 (3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, m, 3, 1+\frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2 \right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] + \right. \right. \\
& \quad \left. \left. \frac{1}{5+m} m (3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1+m, 2, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (c+d x)\right]^2 \right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] - \right. \right. \\
& \quad \left. \left. m \left(-\frac{1}{5+m} (3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1+m, 2, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan\left[\frac{1}{2} (c+d x)\right]^2 \right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] + \frac{1}{5+m} \right. \right. \right. \\
& \quad \left. \left. \left. (1+m) (3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 2+m, 1, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2} (c+d x)\right]^2 \right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] \right) \right) \right) \Big) / \\
& \quad \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan\left[\frac{1}{2} (c+d x)\right]^2 \right] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \tan\left[\frac{1}{2} (c+d x)\right]^2 \right)^2 \right) - \right. \\
& \quad \left. \left(2^{1+n} a b (3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \tan\left[\frac{1}{2} (c+d x)\right]^2 \right)^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^n \\
& \left(-2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2 \right] - m \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2 \right] \right) \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] + \right. \\
& \quad \left. (3+m+n) \left(-\frac{1}{3+m+n}(1+n)(1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), m, 2+n, \right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2 \right] \sec\left[\frac{1}{2}(c+d x)\right]^2 \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(c+d x)\right] + \frac{1}{3+m+n}m(1+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(1+m+n), \right. \right. \right. \\
& \quad \left. \left. \left. 1+m, 1+n, 1+\frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] \right) - 2 \tan\left[\frac{1}{2}(c+d x)\right]^2 \right. \right. \\
& \quad \left((1+n) \left(-\frac{1}{5+m+n}(2+n)(3+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+n), m, \right. \right. \right. \\
& \quad \left. \left. \left. 3+n, 1+\frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] + \frac{1}{5+m+n}m(3+m+n) \operatorname{AppellF1}\left[\right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1}{2}(3+m+n), 1+m, 2+n, 1+\frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2 \right] \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] \right) - \right. \right. \\
& \quad \left. \left. m \left(-\frac{1}{5+m+n}(1+n)(3+m+n) \operatorname{AppellF1}\left[1+\frac{1}{2}(3+m+n), 1+m, \right. \right. \right. \\
& \quad \left. \left. \left. 2+n, 1+\frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] + \frac{1}{5+m+n}(1+m)(3+m+n) \operatorname{AppellF1}\left[\right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{1}{2}(3+m+n), 2+m, 1+n, 1+\frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2 \right] \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] \right) \right) \right) \right) \right) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2] - m \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \right. \\
& \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\Big)^2\Big) - \\
& \left(4^n b^2 (3+m+2 n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+2 n), m, 1+2 n, \frac{1}{2} (3+m+2 n), \right. \right. \\
& \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}\right)^{2 n} \\
& \left(-2 \left((1+2 n) \operatorname{AppellF1}\left[\frac{1}{2} (3+m+2 n), m, 2 (1+n), \frac{1}{2} (5+m+2 n), \right. \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{1}{2} (3+m+2 n), \right. \right. \\
& \left. \left. 1+m, 1+2 n, \frac{1}{2} (5+m+2 n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right]\right) \\
& \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] + (3+m+2 n) \left(-\frac{1}{3+m+2 n} (1+2 n) \right. \\
& \left.(1+m+2 n) \operatorname{AppellF1}\left[1+\frac{1}{2} (1+m+2 n), m, 2+2 n, 1+\frac{1}{2} (3+m+2 n), \right. \right. \\
& \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] + \\
& \left.\frac{1}{3+m+2 n} m (1+m+2 n) \operatorname{AppellF1}\left[1+\frac{1}{2} (1+m+2 n), 1+m, 1+2 n, \right. \right. \\
& \left. \left. 1+\frac{1}{2} (3+m+2 n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right) - 2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \\
& \left((1+2 n) \left(-\frac{1}{5+m+2 n} 2 (1+n) (3+m+2 n) \operatorname{AppellF1}\left[1+\frac{1}{2} (3+m+2 n), m, \right. \right. \right. \\
& \left. \left. 1+2 (1+n), 1+\frac{1}{2} (5+m+2 n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] + \frac{1}{5+m+2 n} m (3+m+2 n) \operatorname{AppellF1}\left[\right. \right. \\
& \left. \left. 1+\frac{1}{2} (3+m+2 n), 1+m, 2 (1+n), 1+\frac{1}{2} (5+m+2 n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \\
& \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right) - \\
& \left.m \left(-\frac{1}{5+m+2 n} (1+2 n) (3+m+2 n) \operatorname{AppellF1}\left[1+\frac{1}{2} (3+m+2 n), 1+m, 2+2 n, \right. \right. \right. \\
& \left. \left. 1+\frac{1}{2} (5+m+2 n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] + \frac{1}{5+m+2 n} (1+m) (3+m+2 n) \operatorname{AppellF1}\left[\right. \right. \right. \\
& \left. \left. \left. 1+\frac{1}{2} (3+m+2 n), 2+m, 1+2 n, 1+\frac{1}{2} (5+m+2 n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\left(1 + m + 2n \right) \left(3 + m + 2n \right) \text{AppellF1} \left[\frac{1}{2} (1 + m + 2n), m, 1 + 2n, \frac{1}{2} (3 + m + 2n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - 2 \left((1 + 2n) \text{AppellF1} \left[\frac{1}{2} (3 + m + 2n), \right. \right. \right. \\
& \quad \left. \left. \left. m, 2 (1 + n), \frac{1}{2} (5 + m + 2n), \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. \left. m \text{AppellF1} \left[\frac{1}{2} (3 + m + 2n), 1 + m, 1 + 2n, \frac{1}{2} (5 + m + 2n), \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \right)
\end{aligned}$$

Problem 576: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b \sin[c + d x]^n) \tan[c + d x]^m dx$$

Optimal (type 5, 124 leaves, 6 steps):

$$\begin{aligned}
& \frac{a \text{Hypergeometric2F1} \left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan[c + d x]^2 \right] \tan[c + d x]^{1+m}}{d (1+m)} + \frac{1}{d (1+m+n)} b (\cos[c + d x]^2)^{\frac{1+m}{2}} \\
& \text{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{1}{2} (1+m+n), \frac{1}{2} (3+m+n), \sin[c + d x]^2 \right] \sin[c + d x]^n \tan[c + d x]^{1+m}
\end{aligned}$$

Result (type 6, 5184 leaves):

$$\begin{aligned}
& \left(2^{1+m} \tan \left[\frac{1}{2} (c + d x) \right] \left(-\frac{\tan \left[\frac{1}{2} (c + d x) \right]}{-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2} \right)^m \right. \\
& \left. \left(a (3 + m) \text{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \right. \\
& \left. \left((1 + m) \left((3 + m) \text{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 \left(\text{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. m \text{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) + \left(2^n b (3 + m + n) \text{AppellF1} \left[\frac{1}{2} (1 + m + n), m, 1 + n, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} (3+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2] \left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2} \right)^n \Bigg) / \\
& \left((1+m+n) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \right. \right. \right. \right. \\
& \left. \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - m \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), 1+m, 1+n, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{1}{2} (5+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right) \Bigg) \\
& (a \operatorname{Tan}[c+d x]^m + b \operatorname{Sin}[c+d x]^n \operatorname{Tan}[c+d x]^m) \Bigg) / \left(d \right. \\
& \left(1 + \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right) \\
& \left(-\frac{1}{\left(1 + \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right)^2} 2^{1+m} \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \left(-\frac{\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2} \right)^m \right. \\
& \left(\left(a (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right) \Bigg) / \right. \\
& \left((1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right) + \\
& \left(2^n b (3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \\
& \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2} \right)^n \right) / \left((1+m+n) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - \frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right) - \\
& 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - m \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{1}{2} \left(c + d x \right)^2, -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right) \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right) + \\
& \frac{1}{1 + \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2} 2^m \sec \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2 \left(-\frac{\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]}{-1 + \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2} \right)^m \\
& \left(\left(a (3+m) \text{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] \right) / \right. \\
& \left((1+m) \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] - \right. \right. \\
& \left. \left. 2 \left(\text{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] - \right. \right. \\
& \left. \left. m \text{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] \right) \right. \\
& \left. \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right) + \left(2^n b (3+m+n) \text{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \right. \right. \\
& \left. \left. \frac{1}{2} (3+m+n), \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right) \left(\frac{\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]}{1 + \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2} \right)^n \right) / \\
& \left((1+m+n) \left((3+m+n) \text{AppellF1} \left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \right. \right. \right. \\
& \left. \left. \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] - \right. \\
& \left. 2 \left((1+n) \text{AppellF1} \left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], \right. \right. \\
& \left. \left. -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] - m \text{AppellF1} \left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \right. \right. \\
& \left. \left. \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right) \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right) + \\
& \frac{1}{1 + \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2} 2^{1+m} m \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \left(-\frac{\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]}{-1 + \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2} \right)^{-1+m} \\
& \left(\frac{\sec \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2 \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2}{\left(-1 + \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2 \right)^2} - \frac{\sec \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2}{2 \left(-1 + \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right]^2 \right)} \right) \\
& \left(a (3+m) \text{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] \right) / \\
& \left((1+m) \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] - \right. \right. \\
& \left. \left. 2 \left(\text{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] - \right. \right. \\
& \left. \left. m \text{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan \left[\frac{1}{2} \left(c + d x \right)^2 \right], -\tan \left[\frac{1}{2} \left(c + d x \right)^2 \right] \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{m AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \\
& \tan\left[\frac{1}{2}(c+d x)\right]^2\right) + \left(2^n b (3+m+n) \text{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \right. \right. \\
& \left. \left. \frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \left(\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2}\right)^n\right) / \\
& \left((1+m+n) \left((3+m+n) \text{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n), \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left((1+n) \text{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - m \text{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n), \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \tan\left[\frac{1}{2}(c+d x)\right]^2\right) \right) + \\
& \frac{1}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} 2^{1+m} \tan\left[\frac{1}{2}(c+d x)\right] \left(-\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{-1+\tan\left[\frac{1}{2}(c+d x)\right]^2}\right)^m \\
& \left(a (3+m) \left(-\frac{1}{3+m} (1+m) \text{AppellF1}\left[1+\frac{1+m}{2}, m, 2, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] + \frac{1}{3+m} \right. \right. \\
& \left. \left. m (1+m) \text{AppellF1}\left[1+\frac{1+m}{2}, 1+m, 1, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right]\right) \right) / \\
& \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left(\text{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\
& \left. \left. m \text{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \tan\left[\frac{1}{2}(c+d x)\right]^2\right) \right) + \\
& \left(2^n b (3+m+n) \left(-\frac{1}{3+m+n} (1+n) (1+m+n) \text{AppellF1}\left[1+\frac{1}{2}(1+m+n), m, 2+n, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \sec\left[\frac{1}{2}(c+d x)\right]^2 \right. \right. \right. \\
& \left. \left. \left. 1+\frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \sec\left[\frac{1}{2}(c+d x)\right]^2\right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\tan\left[\frac{1}{2}(c+d x)\right]}{2} + \frac{1}{3+m+n} m (1+m+n) \text{AppellF1}\left[1+\frac{1}{2}(1+m+n),\right. \\
& \quad \left.1+m, 1+n, 1+\frac{1}{2}(3+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \\
& \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] \left(\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^n \Bigg) / \\
& \left((1+m+n) \left((3+m+n) \text{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n),\right.\right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - m \text{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n),\right.\right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+d x)\right]^2 \Bigg) + \\
& \left(2^n b n (3+m+n) \text{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n),\right.\right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \left(\frac{\tan\left[\frac{1}{2}(c+d x)\right]}{1+\tan\left[\frac{1}{2}(c+d x)\right]^2} \right)^{-1+n} \\
& \left(-\frac{\sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right]^2}{\left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2\right)^2} + \frac{\sec\left[\frac{1}{2}(c+d x)\right]^2}{2 \left(1+\tan\left[\frac{1}{2}(c+d x)\right]^2\right)} \right) \Bigg) / \\
& \left((1+m+n) \left((3+m+n) \text{AppellF1}\left[\frac{1}{2}(1+m+n), m, 1+n, \frac{1}{2}(3+m+n),\right.\right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((1+n) \text{AppellF1}\left[\frac{1}{2}(3+m+n), m, 2+n, \frac{1}{2}(5+m+n), \tan\left[\frac{1}{2}(c+d x)\right]^2,\right.\right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - m \text{AppellF1}\left[\frac{1}{2}(3+m+n), 1+m, 1+n, \frac{1}{2}(5+m+n),\right.\right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+d x)\right]^2 \right) - \\
& \left(a (3+m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right. \\
& \quad \left(-2 \left(\text{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\
& \quad \left. \left. m \text{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right) \right. \\
& \quad \left. \sec\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] + (3+m) \left(-\frac{1}{3+m} (1+m) \text{AppellF1}\left[1+\frac{1+m}{2},\right.\right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \right)
\end{aligned}$$

$$\begin{aligned}
 & m, 2, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2 \sec\left[\frac{1}{2} (c+d x)\right]^2 \\
 & \tan\left[\frac{1}{2} (c+d x)\right] + \frac{1}{3+m} m (1+m) \text{AppellF1}\left[1 + \frac{1+m}{2}, 1+m, 1, 1 + \frac{3+m}{2}, \right. \\
 & \left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] - \\
 & 2 \tan\left[\frac{1}{2} (c+d x)\right]^2 \left(-\frac{1}{5+m} 2 (3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, m, 3, 1 + \frac{5+m}{2}, \right.\right. \\
 & \left.\left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] + \right. \\
 & \left. \frac{1}{5+m} m (3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 1+m, 2, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right.\right. \\
 & \left.\left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] - \right. \\
 & \left. m \left(-\frac{1}{5+m} (3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 1+m, 2, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right.\right. \right. \\
 & \left.\left.\left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] + \frac{1}{5+m} \right. \\
 & \left. (1+m) (3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 2+m, 1, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right.\right. \\
 & \left.\left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right]\right)\right)\right) \\
 & \left((1+m) \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right.\right. \right. \\
 & \left.\left.\left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - 2 \left(\text{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2} (c+d x)\right]^2, \right.\right. \right.\right. \\
 & \left.\left.\left.\left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - m \text{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \right.\right. \right.\right. \\
 & \left.\left.\left.\left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right]\right) \tan\left[\frac{1}{2} (c+d x)\right]^2\right)^2\right) - \\
 & \left(2^n b (3+m+n) \text{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \tan\left[\frac{1}{2} (c+d x)\right]^2, \right.\right. \\
 & \left.\left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \left(\frac{\tan\left[\frac{1}{2} (c+d x)\right]}{1+\tan\left[\frac{1}{2} (c+d x)\right]^2}\right)^n \right. \\
 & \left.-2 \left((1+n) \text{AppellF1}\left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \tan\left[\frac{1}{2} (c+d x)\right]^2, \right.\right. \right. \\
 & \left.\left.\left. -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] - m \text{AppellF1}\left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \right.\right. \right. \\
 & \left.\left.\left. \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right]\right) \sec\left[\frac{1}{2} (c+d x)\right]^2 \tan\left[\frac{1}{2} (c+d x)\right] + \right. \\
 & \left.(3+m+n) \left(-\frac{1}{3+m+n} (1+n) (1+m+n) \text{AppellF1}\left[1 + \frac{1}{2} (1+m+n), m, 2+n, \right.\right. \right. \\
 & \left.\left.\left. 1 + \frac{1}{2} (3+m+n), \tan\left[\frac{1}{2} (c+d x)\right]^2, -\tan\left[\frac{1}{2} (c+d x)\right]^2\right] \sec\left[\frac{1}{2} (c+d x)\right]^2 \right)
 \end{aligned}$$

$$\begin{aligned}
& \left. \left(\left(\frac{1}{2} \operatorname{Tan}[c + d x] + \frac{1}{3+m+n} m (1+m+n) \operatorname{AppellF1}\left[1 + \frac{1}{2} (1+m+n), \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 1+m, 1+n, 1+\frac{1}{2} (3+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right. \right. \\
& \quad \left. \left. \left. \left. \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] \right) - 2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right. \right. \\
& \quad \left. \left. \left. \left. \left((1+n) \left(-\frac{1}{5+m+n} (2+n) (3+m+n) \operatorname{AppellF1}\left[1 + \frac{1}{2} (3+m+n), m, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. 3+n, 1+\frac{1}{2} (5+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] + \frac{1}{5+m+n} m (3+m+n) \operatorname{AppellF1}\left[\right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. 1+\frac{1}{2} (3+m+n), 1+m, 2+n, 1+\frac{1}{2} (5+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] \right) - \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. m \left(-\frac{1}{5+m+n} (1+n) (3+m+n) \operatorname{AppellF1}\left[1 + \frac{1}{2} (3+m+n), 1+m, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. 2+n, 1+\frac{1}{2} (5+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] + \frac{1}{5+m+n} (1+m) (3+m+n) \operatorname{AppellF1}\left[\right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. 1+\frac{1}{2} (3+m+n), 2+m, 1+n, 1+\frac{1}{2} (5+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right] \right) \right) \right) \right) \right) \right) \right) / \\
& \quad \left. \left(\left(1+m+n \right) \left((3+m+n) \operatorname{AppellF1}\left[\frac{1}{2} (1+m+n), m, 1+n, \frac{1}{2} (3+m+n), \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 \left((1+n) \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), m, 2+n, \frac{1}{2} (5+m+n), \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right] - m \operatorname{AppellF1}\left[\frac{1}{2} (3+m+n), 1+m, 1+n, \frac{1}{2} (5+m+n), \right. \right. \right. \\
& \quad \left. \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 585: Unable to integrate problem.

$$\int \operatorname{Cot}[c+d x]^3 (a+b \sin[c+d x]^n)^p dx$$

Optimal (type 5, 136 leaves, 7 steps):

$$\frac{1}{a d n (1+p)} \text{Hypergeometric2F1}\left[1, 1+p, 2+p, 1 + \frac{b \sin[c+d x]^n}{a}\right] (a + b \sin[c+d x]^n)^{1+p} - \\ \frac{1}{2 d} \csc[c+d x]^2 \text{Hypergeometric2F1}\left[-\frac{2}{n}, -p, -\frac{2-n}{n}, -\frac{b \sin[c+d x]^n}{a}\right] \\ (a + b \sin[c+d x]^n)^p \left(1 + \frac{b \sin[c+d x]^n}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \cot[c+d x]^3 (a + b \sin[c+d x]^n)^p dx$$

Problem 591: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \sin[e+f x]^2}{(g \cos[e+f x])^{5/2} \sqrt{d \sin[e+f x]}} dx$$

Optimal (type 4, 107 leaves, 7 steps):

$$\frac{2 (a+b) \sqrt{d \sin[e+f x]}}{3 d f g (g \cos[e+f x])^{3/2}} + \frac{(2 a-b) \text{EllipticF}\left[e-\frac{\pi}{4}+f x, 2\right] \sqrt{\sin[2 e+2 f x]}}{3 f g^2 \sqrt{g \cos[e+f x]} \sqrt{d \sin[e+f x]}}$$

Result (type 5, 120 leaves):

$$\left(2 \left(-2 (2 a-b) \cos[e+f x]^2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[e+f x]^2\right] + (a+b+(2 a-b) \cos[e+f x]^2) (\sin[e+f x]^2)^{1/4}\right) \tan[e+f x]\right) / \\ (3 f g^2 \sqrt{g \cos[e+f x]} \sqrt{d \sin[e+f x]} (\sin[e+f x]^2)^{1/4})$$

Problem 592: Result more than twice size of optimal antiderivative.

$$\int (c \cos[e+f x])^m (d \sin[e+f x])^n (a + b \sin[e+f x]^2)^p dx$$

Optimal (type 6, 137 leaves, 3 steps):

$$\frac{1}{d f (1+n)} c \text{AppellF1}\left[\frac{1+n}{2}, \frac{1-m}{2}, -p, \frac{3+n}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] (c \cos[e+f x])^{-1+m} \\ (\cos[e+f x]^2)^{\frac{1-m}{2}} (d \sin[e+f x])^{1+n} (a + b \sin[e+f x]^2)^p \left(1 + \frac{b \sin[e+f x]^2}{a}\right)^{-p}$$

Result (type 6, 279 leaves):

$$\left(\frac{a (3+n) \text{AppellF1}\left[\frac{1+n}{2}, \frac{1-m}{2}, -p, \frac{3+n}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right]}{(c \cos[e+f x])^m (d \sin[e+f x])^n (a+b \sin[e+f x]^2)^p \tan[e+f x]} \right) / \\ \left(f (1+n) \left(a (3+n) \text{AppellF1}\left[\frac{1+n}{2}, \frac{1-m}{2}, -p, \frac{3+n}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] + \right. \right. \\ \left. \left. \left(2 b p \text{AppellF1}\left[\frac{3+n}{2}, \frac{1-m}{2}, 1-p, \frac{5+n}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right] - a (-1+m) \right. \right. \right. \\ \left. \left. \left. \text{AppellF1}\left[\frac{3+n}{2}, \frac{3-m}{2}, -p, \frac{5+n}{2}, \sin[e+f x]^2, -\frac{b \sin[e+f x]^2}{a}\right]\right) \sin[e+f x]^2 \right) \right)$$

Problem 593: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + (c \cos[e+f x] + b \sin[e+f x])^2} dx$$

Optimal (type 4, 79 leaves, 3 steps):

$$\left(\text{EllipticE}\left[e+f x+\text{ArcTan}[b, c], -\frac{b^2+c^2}{a}\right] \sqrt{a + (c \cos[e+f x] + b \sin[e+f x])^2} \right) / \\ \left(f \sqrt{1 + \frac{(c \cos[e+f x] + b \sin[e+f x])^2}{a}} \right)$$

Result (type 4, 325 leaves):

$$\begin{aligned}
& - \left(\left[\frac{\text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{\frac{\sqrt{(b^2+c^2)^2+(b^2-c^2) \cos[2(e+f x)]-2 b c \sin[2(e+f x)]}{\sqrt{(b^2+c^2)^2}}}{\sqrt{2}}\right], \frac{2 \sqrt{(b^2+c^2)^2}}{2 a+b^2+c^2+\sqrt{(b^2+c^2)^2}}]} \right. \right. \\
& \quad \left. \left. \left(2 b c \cos[2(e+f x)] + (b^2-c^2) \sin[2(e+f x)] \right) \right] \right. \\
& \quad \left. \left(\sqrt{2} \sqrt{(b^2+c^2)^2} f \sqrt{\left((2 a+b^2+c^2+(-b^2+c^2) \cos[2(e+f x)]+2 b c \sin[2(e+f x)]) \right.} \right. \\
& \quad \left. \left. \left. \left(2 a+b^2+c^2+\sqrt{(b^2+c^2)^2} \right) \right) \sqrt{\frac{(2 b c \cos[2(e+f x)] + (b^2-c^2) \sin[2(e+f x)])^2}{(b^2+c^2)^2}} \right) \right)
\end{aligned}$$

Problem 594: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + (c \cos[e+f x] + b \sin[e+f x])^2}} dx$$

Optimal (type 4, 79 leaves, 3 steps):

$$\left(\left[\text{EllipticF}[e+f x + \text{ArcTan}[b, c], -\frac{b^2+c^2}{a}] \sqrt{1 + \frac{(c \cos[e+f x] + b \sin[e+f x])^2}{a}} \right] \right. \\
\left. \left(f \sqrt{a + (c \cos[e+f x] + b \sin[e+f x])^2} \right) \right)$$

Result (type 6, 529 leaves):

$$\frac{1}{b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} f}$$

$$\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \sin [2 (e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]]}{2 a+b^2+c^2-b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}}},\right.$$

$$\left.\frac{2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \sin [2 (e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]]}{2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}}}\right]$$

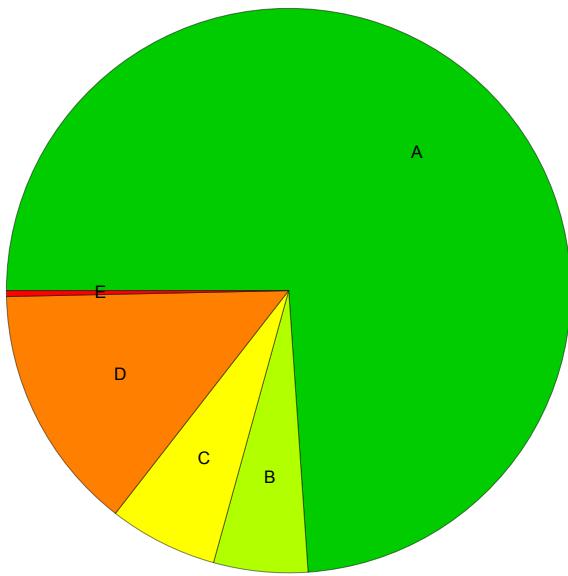
$$\sec \left[2 (e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right] \sqrt{-\frac{b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \left(-1+\sin \left[2 (e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right]\right)}{2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}}}}$$

$$\sqrt{-\frac{b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \left(1+\sin \left[2 (e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right]\right)}{2 a+b^2+c^2-b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}}}}$$

$$\sqrt{\left(2 a+b^2+c^2+b c \sqrt{\frac{(b^2+c^2)^2}{b^2 c^2}} \sin \left[2 (e+f x)+\operatorname{ArcTan}\left[\frac{-b^2+c^2}{2 b c}\right]\right]\right)}$$

Summary of Integration Test Results

594 integration problems



A - 439 optimal antiderivatives

B - 32 more than twice size of optimal antiderivatives

C - 37 unnecessarily complex antiderivatives

D - 84 unable to integrate problems

E - 2 integration timeouts